HOMEWOR II

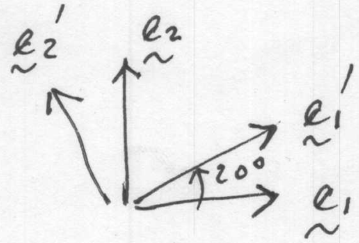
(1) CONSIDER THE SECOND-ORDER TENSOR $\mathbf{T} = \mathbf{e}_1\mathbf{e}_1 + 4\mathbf{e}_1\mathbf{e}_2 + 4\mathbf{e}_2\mathbf{e}_1 + 7\mathbf{e}_2\mathbf{e}_2$

THE COMPONENTS OF THIS TENSOR IN THE SYSTEM $(\mathbf{e}_1, \mathbf{e}_2)$ ARE $T_{11} = 1$, $T_{12} = T_{21} = 4$, AND $T_{22} = 7$.

a) BY OPERATING WITH THE COMPONENTS T_{ij}, OBTAIN THE EIGENVALUES AND THE ATTENDANT EIGENVECTORS.
FOR YOU TO CHECK: $\lambda_1 = 9$, $\mathbf{v}_1 = \mathbf{e}_1 + 2\mathbf{e}_2$
$\lambda_2 = -1$, $\mathbf{v}_2 = -2\mathbf{e}_1 + \mathbf{e}_2$.

VERIFY THAT $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$. WHAT DOES THIS MEAN?

b) CONSIDER NOW A SYSTEM $(\mathbf{e}_1', \mathbf{e}_2')$ ROTATED 20° FROM $(\mathbf{e}_1, \mathbf{e}_2)$. WRITE AN EXPRESSION FOR $\mathbf{e}_1'$ AND AN EXPRESSION FOR $\mathbf{e}_2'$ IN TERMS OF $\mathbf{e}_1$ AND $\mathbf{e}_2$. FOR EXAMPLE,



$$\mathbf{e}_1' = 0.9397\,\mathbf{e}_1 + 0.3420\,\mathbf{e}_2$$

c) BY USING THE EXPRESSION $T_{ij}' = \mathbf{e}_i' \cdot \mathbf{T} \cdot \mathbf{e}_j'$, OBTAIN THE COMPONENTS OF $\mathbf{T}$ IN THE SYSTEM $(\mathbf{e}_1', \mathbf{e}_2')$. YOU SHOULD BE ABLE TO CONCLUDE THAT

$$\mathbf{T} = 4.2730\,\mathbf{e}_1'\mathbf{e}_1' + 4.9925\,\mathbf{e}_1'\mathbf{e}_2' + 4.9925\,\mathbf{e}_2'\mathbf{e}_1' + 3.7270\,\mathbf{e}_2'\mathbf{e}_2'$$

NOTE THAT THE SYMMETRY IS AN INVARIANT PROPERTY THAT PERTAINS TO THE TENSOR AND MANIFESTS ITSELF IN ALL CARTESIAN SYSTEMS, AS EXPECTED.