

Dundurs' Catalog for Polar Components of Stress and Displacement

$$\kappa = 3 - 4\nu$$

$$\kappa = \frac{3 - \nu}{1 + \nu}$$

For plane strain

For plane stress

Rigid body displacements:

$$u_r = C_1 \cos(\theta) + C_2 \sin(\theta), \quad u_\theta = -C_1 \sin(\theta) + C_2 \cos(\theta) + C_3 r$$

U	σ_{rr}	$\sigma_{r\theta}$	$\sigma_{\theta\theta}$	$2Gu_r$	$2Gu_\theta$
r^2	2	0	2	$(\kappa - 1)r$	0
$\log(r)$	$\frac{1}{r^2}$	0	$-\frac{1}{r^2}$	$-\frac{1}{r}$	0
θ	0	$\frac{1}{r^2}$	0	0	$-\frac{1}{r}$
$r^2 \log(r)$	$2 \log(r) + 1$	0	$2 \log(r) + 3$	$(\kappa - 1)r \log(r) - r$	$(\kappa + 1)r\theta$
$r^2 \theta$	2θ	-1	2θ	$(\kappa - 1)r\theta$	$-(\kappa + 1)r \log(r)$
$r^3 \cos(\theta)$	$2r \cos(\theta)$	$2r \sin(\theta)$	$6r \cos(\theta)$	$(\kappa - 2)r^2 \cos(\theta)$	$(\kappa + 2)r^2 \sin(\theta)$
$r^3 \sin(\theta)$	$2r \sin(\theta)$	$-2r \cos(\theta)$	$6r \sin(\theta)$	$(\kappa - 2)r^2 \sin(\theta)$	$-(\kappa + 2)r^2 \cos(\theta)$
$r\theta \sin(\theta)$	$\frac{2 \cos(\theta)}{r}$	0	0	$\frac{(\kappa - 1)\theta \sin(\theta) + (\kappa + 1) \log(r) \cos(\theta) - \cos(\theta)}{2}$	$\frac{(\kappa - 1)\theta \cos(\theta) - (\kappa + 1) \log(r) \sin(\theta) - \sin(\theta)}{2}$
$r\theta \cos(\theta)$	$\frac{-2 \sin(\theta)}{r}$	0	0	$\frac{(\kappa - 1)\theta \cos(\theta) - (\kappa + 1) \log(r) \sin(\theta) + \sin(\theta)}{2}$	$\frac{-(\kappa - 1)\theta \sin(\theta) - (\kappa + 1) \log(r) \cos(\theta) - \cos(\theta)}{2}$
$r \log(r) \cos(\theta)$	$\frac{\cos(\theta)}{r}$	$\frac{\sin(\theta)}{r}$	$\frac{\cos(\theta)}{r}$	$\frac{(\kappa + 1)\theta \sin(\theta) + (\kappa - 1) \log(r) \cos(\theta) - \cos(\theta)}{2}$	$\frac{(\kappa + 1)\theta \cos(\theta) - (\kappa - 1) \log(r) \sin(\theta) - \sin(\theta)}{2}$
$r \log(r) \sin(\theta)$	$\frac{\sin(\theta)}{r}$	$\frac{-\cos(\theta)}{r}$	$\frac{\sin(\theta)}{r}$	$\frac{-(\kappa + 1)\theta \cos(\theta) + (\kappa - 1) \log(r) \sin(\theta) - \sin(\theta)}{2}$	$\frac{(\kappa + 1)\theta \sin(\theta) + (\kappa - 1) \log(r) \cos(\theta) + \cos(\theta)}{2}$
$\frac{\cos(\theta)}{r}$	$\frac{-2 \cos(\theta)}{r^3}$	$\frac{-2 \sin(\theta)}{r^3}$	$\frac{2 \cos(\theta)}{r^3}$	$\frac{\cos(\theta)}{r^2}$	$\frac{\sin(\theta)}{r^2}$
$\frac{\sin(\theta)}{r}$	$\frac{-2 \sin(\theta)}{r^3}$	$\frac{2 \cos(\theta)}{r^3}$	$\frac{2 \sin(\theta)}{r^3}$	$\frac{\sin(\theta)}{r^2}$	$\frac{-\cos(\theta)}{r^2}$

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For plane stress

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$$u_r = C_1 \cos(\theta) + C_2 \sin(\theta), \quad u_\theta = -C_1 \sin(\theta) + C_2 \cos(\theta) + C_3 r$$

U	σ_{rr}	$\sigma_{r\theta}$	$\sigma_{\theta\theta}$	$2Gu_r$	$2Gu_\theta$
$r^2 \cos(2\theta)$	$-2 \cos(2\theta)$	$2 \sin(2\theta)$	$2 \cos(2\theta)$	$-2r \cos(2\theta)$	$2r \sin(2\theta)$
$r^2 \sin(2\theta)$	$-2 \sin(2\theta)$	$-2 \cos(2\theta)$	$2 \sin(2\theta)$	$-2r \sin(2\theta)$	$-2r \cos(2\theta)$
$r^4 \cos(2\theta)$	0	$6r^2 \sin(2\theta)$	$12r^2 \cos(2\theta)$	$-(3 - \kappa)r^3 \cos(2\theta)$	$(3 + \kappa)r^3 \sin(2\theta)$
$r^4 \sin(2\theta)$	0	$-6r^2 \cos(2\theta)$	$12r^2 \sin(2\theta)$	$-(3 - \kappa)r^3 \sin(2\theta)$	$-(3 + \kappa)r^3 \cos(2\theta)$
$\frac{\cos(2\theta)}{r^2}$	$-\frac{6 \cos(2\theta)}{r^4}$	$-\frac{6 \sin(2\theta)}{r^4}$	$\frac{6 \cos(2\theta)}{r^4}$	$\frac{2 \cos(2\theta)}{r^3}$	$\frac{2 \sin(2\theta)}{r^3}$
$\frac{\sin(2\theta)}{r^2}$	$-\frac{6 \sin(2\theta)}{r^4}$	$\frac{6 \cos(2\theta)}{r^4}$	$\frac{6 \sin(2\theta)}{r^4}$	$\frac{2 \sin(2\theta)}{r^3}$	$-\frac{2 \cos(2\theta)}{r^3}$
$\cos(2\theta)$	$-\frac{4 \cos(2\theta)}{r^2}$	$-\frac{2 \sin(2\theta)}{r^2}$	$-\frac{2 \sin(2\theta)}{r^2}$	$\frac{(\kappa + 1) \cos(2\theta)}{r}$	$-\frac{(\kappa - 1) \sin(2\theta)}{r}$
$\sin(2\theta)$	$-\frac{4 \sin(2\theta)}{r^2}$	$\frac{2 \cos(2\theta)}{r^2}$	0	$\frac{(\kappa + 1) \sin(2\theta)}{r}$	$\frac{(\kappa - 1) \cos(2\theta)}{r}$
$r^n \cos(n\theta)$	$-n(n-1)r^{n-2} \cos(n\theta)$	$n(n-1)r^{n-2} \sin(n\theta)$	$n(n-1)r^{n-2} \cos(n\theta)$	$-nr^{n-1} \cos(n\theta)$	$nr^{n-1} \sin(n\theta)$
$r^n \sin(n\theta)$	$-n(n-1)r^{n-2} \sin(n\theta)$	$-n(n-1)r^{n-2} \cos(n\theta)$	$n(n-1)r^{n-2} \sin(n\theta)$	$-nr^{n-1} \sin(n\theta)$	$-nr^{n-1} \cos(n\theta)$
$r^{n+2} \cos(n\theta)$	$-(n+1)(n-2)r^n \cos(n\theta)$	$n(n+1)r^n \sin(n\theta)$	$(n+2)(n+1)r^n \cos(n\theta)$	$-(n+1 - \kappa)r^{n+1} \cos(n\theta)$	$(n+1 + \kappa)r^{n+1} \sin(n\theta)$
$r^{n+2} \sin(n\theta)$	$-(n+1)(n-2)r^n \sin(n\theta)$	$-n(n+1)r^n \cos(n\theta)$	$(n+2)(n+1)r^n \sin(n\theta)$	$-(n+1 - \kappa)r^{n+1} \sin(n\theta)$	$-(n+1 + \kappa)r^{n+1} \cos(n\theta)$
$\frac{\cos(n\theta)}{r^n}$	$-\frac{n(n+1) \cos(n\theta)}{r^{n+2}}$	$-\frac{n(n+1) \sin(n\theta)}{r^{n+2}}$	$\frac{n(n+1) \cos(n\theta)}{r^{n+2}}$	$\frac{n \cos(n\theta)}{r^{n+1}}$	$\frac{n \sin(n\theta)}{r^{n+1}}$
$\frac{\sin(n\theta)}{r^n}$	$-\frac{n(n+1) \sin(n\theta)}{r^{n+2}}$	$\frac{n(n+1) \cos(n\theta)}{r^{n+2}}$	$\frac{n(n+1) \sin(n\theta)}{r^{n+2}}$	$\frac{n \sin(n\theta)}{r^{n+1}}$	$-\frac{n \cos(n\theta)}{r^{n+1}}$
$\frac{\cos(n\theta)}{r^{n-2}}$	$-\frac{(n+2)(n-1) \cos(n\theta)}{r^n}$	$-\frac{(n+2)(n-1) \sin(n\theta)}{r^n}$	$-\frac{(n+2)(n-1) \cos(n\theta)}{r^n}$	$\frac{(n-1 + \kappa) \cos(n\theta)}{r^{n-1}}$	$\frac{(n-1 - \kappa) \sin(n\theta)}{r^{n-1}}$
$\frac{\sin(n\theta)}{r^{n-2}}$	$-\frac{(n+2)(n-1) \sin(n\theta)}{r^n}$	$\frac{n(n-1) \cos(n\theta)}{r^n}$	$\frac{(n-1)(n-2) \sin(n\theta)}{r^n}$	$\frac{(n-1 + \kappa) \sin(n\theta)}{r^{n-1}}$	$-\frac{(n-1 - \kappa) \cos(n\theta)}{r^{n-1}}$

Airy Stress Functions & Their Physical Interpretations

Airy Stress Function (U)	Comments	Singularities
r^2	Field of Uniform biaxial stress	
$\log(r)$	Center of Dilatation	S^0
θ	Concentrated Couple	S^0
$r^2 \log(r)$	Wedge Disclination	S^0, S^∞
$r^2 \theta$	Somigliana Dislocation + Line Load	
$r^2 \cos(\theta)$	Linearly Varying Stress Field	S^∞
$r^2 \sin(\theta)$	Linearly Varying Stress Field	S^∞
$r\theta \sin(\theta)$	Concentrated Force	S^0
$r\theta \cos(\theta)$	Concentrated Force	S^0
$r \log(r) \cos(\theta)$	Edge Dislocation	S^0
$r \log(r) \sin(\theta)$	Edge Dislocation	S^0
$\frac{\cos(\theta)}{r}$	Higher Order	
$\frac{\sin(\theta)}{r}$	Higher Order	
$r^2 \cos(2\theta)$	Combination of Uniform Tension, Compression and Shear	
$r^2 \sin(2\theta)$	Combination of Uniform Tension, Compression and Shear	
$\cos(2\theta)$	Centers of Shear	S^0
$\sin(2\theta)$	Centers of Shear	S^0

S^0 - Stress singularity at origin
 S^∞ - Stress singularity at infinity

CONCENTRATED FORCES AND COUPLES

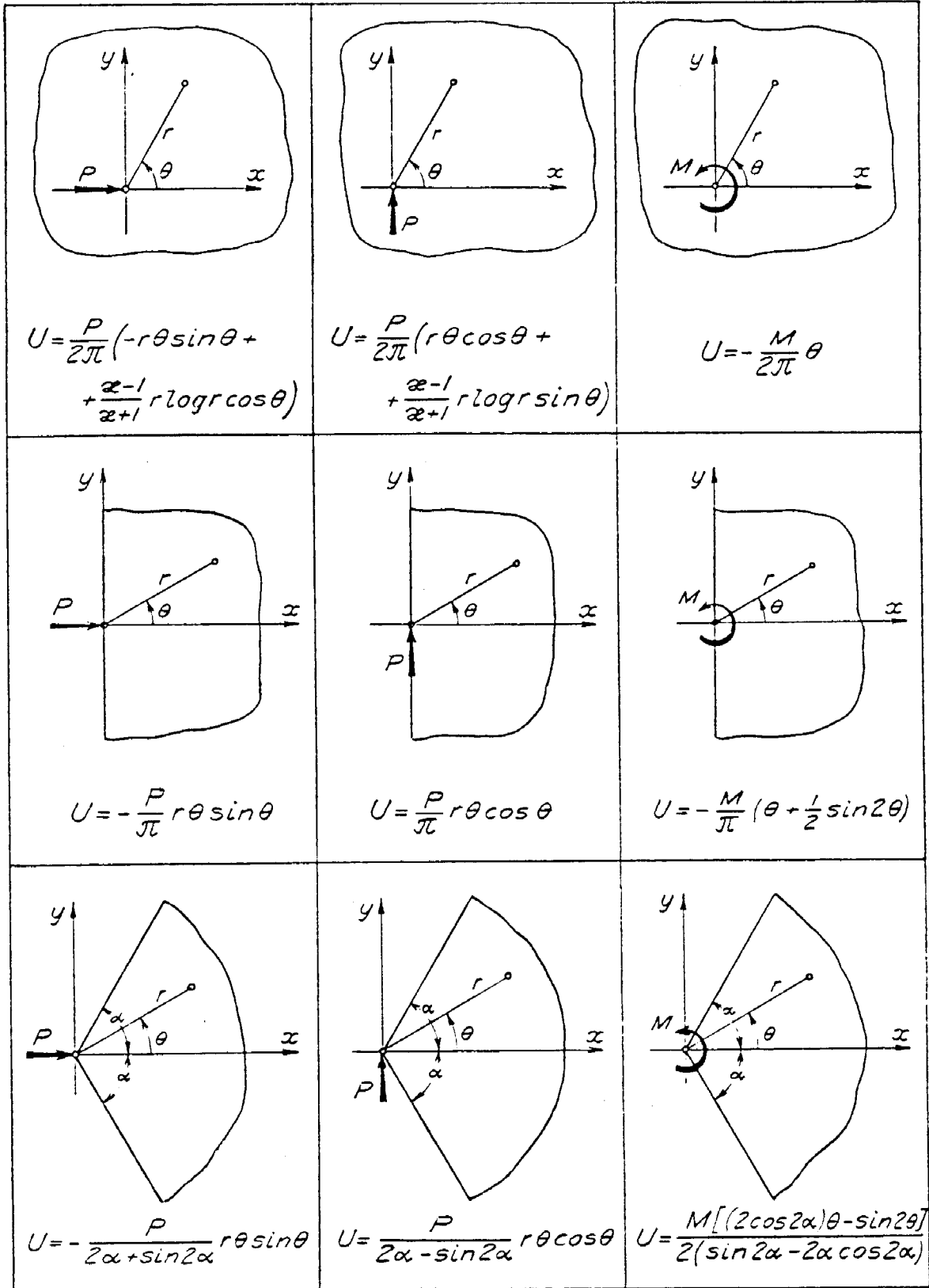


Fig. 1.

antisymmetric

	$\varphi(z)$	$\psi(z)$	U	$2\mu u_r$	$2\mu u_\theta$
A	$z^{1+\lambda}$	0	$r^{2+\lambda} \cos \lambda \theta$	$(\lambda-1-\lambda)r^{1+\lambda} \cos \lambda \theta$	$(\lambda+1+\lambda)r^{1+\lambda} \sin \lambda \theta$
B	0	$z^{1+\lambda}$	$\frac{1}{2+\lambda} r^{2+\lambda} \cos(2+\lambda)\theta$	$-r^{1+\lambda} \cos(2+\lambda)\theta$	$r^{1+\lambda} \sin(2+\lambda)\theta$
C	$iz^{1+\lambda}$	0	$-r^{2+\lambda} \sin \lambda \theta$	$-(\lambda-1-\lambda)r^{1+\lambda} \sin \lambda \theta$	$(\lambda+1+\lambda)r^{1+\lambda} \cos \lambda \theta$
D	0	$iz^{1+\lambda}$	$-\frac{1}{2+\lambda} r^{2+\lambda} \sin(2+\lambda)\theta$	$r^{1+\lambda} \sin(2+\lambda)\theta$	$r^{1+\lambda} \cos(2+\lambda)\theta$

	σ_{rr}	$\sigma_{\theta\theta}$	$\sigma_{r\theta}$	$\sigma_{\theta r}$
A	$(1+\lambda)(2-\lambda)r^\lambda \cos \lambda \theta$	$(1+\lambda)(2+\lambda)r^\lambda \cos \lambda \theta$	$\lambda(1+\lambda)r^\lambda \sin \lambda \theta$	$(1+\lambda)(2-\lambda)r^\lambda \cos \lambda \theta$
B	$-(1+\lambda)r^\lambda \cos(2+\lambda)\theta$	$(1+\lambda)r^\lambda \cos(2+\lambda)\theta$	$(1+\lambda)r^\lambda \sin(2+\lambda)\theta$	$(1+\lambda)r^\lambda \cos(2+\lambda)\theta$
C	$-(1+\lambda)(2-\lambda)r^\lambda \sin \lambda \theta$	$-(1+\lambda)(2+\lambda)r^\lambda \sin \lambda \theta$	$\lambda(1+\lambda)r^\lambda \cos \lambda \theta$	$-(1+\lambda)(2-\lambda)r^\lambda \sin \lambda \theta$
D	$(1+\lambda)r^\lambda \sin(2+\lambda)\theta$	$-(1+\lambda)r^\lambda \sin(2+\lambda)\theta$	$(1+\lambda)r^\lambda \cos(2+\lambda)\theta$	$-(1+\lambda)r^\lambda \sin(2+\lambda)\theta$