

# DISLOCATIONS

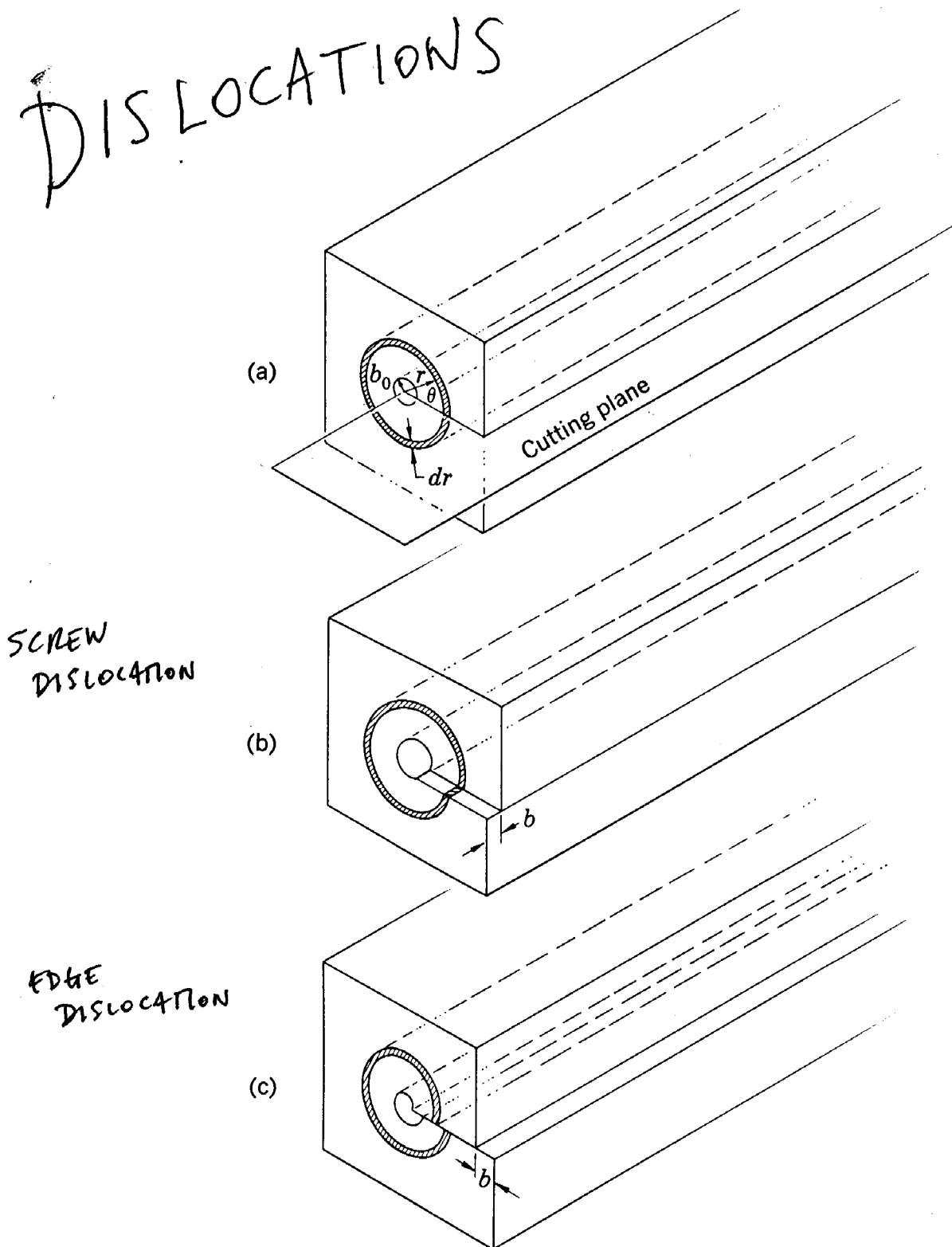


Fig. 10-9. Volterra dislocations in a multiply connected elastic cylinder, i.e., a body that can be cut through without being divided into two pieces. The internal stress fields produced by (a) cutting the cylinder along its axis and rigidly displacing the new faces (b) parallel and (c) perpendicular to the axis, and then rejoining the faces so that the body is continuous at the cut, are respectively identical with the elastic stress fields of a screw dislocation and of an edge dislocation.

# ENERGY (OR SELF-ENERGY) OF A SINGLE DISLOCATION LINE IN PLANE STRAIN

## 1) EDGE DISLOCATION

$$\sigma_{\theta\theta} = \sigma_{rr} = \frac{-2b\mu}{\pi(\lambda+1)} \frac{\cos\theta}{r} = \frac{-b\mu}{2\pi(1-\nu)} \frac{\cos\theta}{r}$$

$$\sigma_{\theta r} = \frac{-2b\mu}{\pi(\lambda+1)} \frac{\sin\theta}{r} = \frac{-b\mu}{2\pi(1-\nu)} \frac{\sin\theta}{r}$$



ENERGY DENSITY  $\mathcal{E} = \frac{1}{2} \sigma_{ij} \epsilon_{ij} = \frac{1}{2} \sigma_{\theta\theta} \epsilon_{\theta\theta} + \frac{1}{2} \sigma_{rr} \epsilon_{rr} + \frac{1}{2} \sigma_{r\theta} \epsilon_{r\theta} + \frac{1}{2} \sigma_{\theta r} \epsilon_{\theta r}$

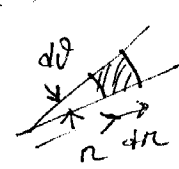
$$\epsilon_{zz} = 0 = \frac{\sigma_{zz}}{E} - \nu \frac{\sigma_{rr}}{E} - \nu \frac{\sigma_{\theta\theta}}{E} \Rightarrow \left( \frac{\sigma_{zz}}{E} \right) = \frac{2\nu}{E} \sigma_{rr}$$

$$\epsilon_{rr} = \frac{\sigma_{rr}}{E} - \nu \frac{\sigma_{\theta\theta}}{E} - \nu \frac{\sigma_{zz}}{E} = \frac{(1-\nu)(1-2\nu)}{E} \sigma_{rr} = \frac{1-2\nu}{2\mu} \sigma_{rr} \quad (E = 2\mu(1+\nu))$$

$$\epsilon_{\theta\theta} = \epsilon_{rr}, \quad \epsilon_{r\theta} = \epsilon_{\theta r} = \frac{\sigma_{r\theta}}{2\mu}$$

$$\begin{aligned} \text{So, } \mathcal{E} &= \sigma_{rr} \epsilon_{rr} + \sigma_{r\theta} \epsilon_{r\theta} = \frac{1-2\nu}{2\mu} \sigma_{rr}^2 + \frac{1}{2\mu} \sigma_{r\theta}^2 \\ &= \frac{1-2\nu}{2\mu} \frac{b^2 \mu^2}{(2\pi)^2 (1-\nu)^2} \frac{\cos^2\theta}{r^2} + \frac{1}{2\mu} \frac{b^2 \mu^2}{(2\pi)^2 (1-\nu)^2} \frac{\sin^2\theta}{r^2} \end{aligned}$$

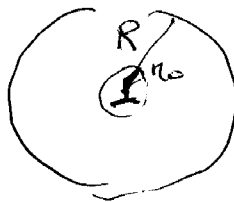
$$T_{\text{EDGE}} = \int_{r=r_0}^{r=R} \int_{\theta=0}^{\theta=2\pi} \mathcal{E} \frac{r d\theta dr}{d\text{Volume}}$$



BUT  $\int_0^{2\pi} \cos^2\theta d\theta = \int_0^{2\pi} \sin^2\theta d\theta = \pi$

$$\text{So, } T_E = \frac{1-2\nu+1}{2 \cdot 4\pi (1-\nu)^2} \int_{r=r_0}^{r=R} \frac{dr}{r} = \frac{b^2 \mu}{4\pi(1-\nu)} \ln\left(\frac{R}{r_0}\right)$$

$$T_E = \frac{b^2 \mu}{4\pi(1-\nu)} \ln\left(\frac{R}{r_0}\right)$$



$r_0 = \text{DISLOCATION CORE}$   
 $R \approx \text{SIZE BODY}$   
 IF  $r_0$  OR  $R \rightarrow 0$ ,  $T_E \rightarrow \infty$

## 2) SCREW DISLOCATION

SEE HOMEWORK

$$T_S = \frac{b^2 \mu}{4\pi} \ln\left(\frac{R}{r_0}\right)$$

### 3.2 THE YIELD STRENGTH OF A PERFECT CRYSTAL

The yield strength of a "perfect" crystal is considerably greater than the stresses commonly associated with plastic flow initiation in most crystalline solids. For example, a reasonably sized pure single crystal of tungsten, the most refractory of the metals, can be bent permanently by hand. The dichotomy between "theory" and experiment is resolved by the dislocation concept. But we first consider the problem of just how strong a perfect crystal should be.

To estimate this, we assume that atomic sliding on slip planes is the cause of plastic flow. Such an ideal shearing process is illustrated schematically in Fig. 3.1a. For the atoms in the upper row (plane) to slide over those in the lower one, strong interatomic forces must be overcome by the applied stress. The variation in crystal

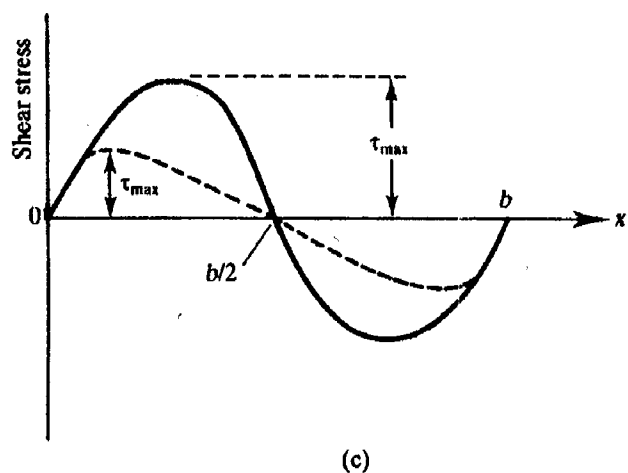
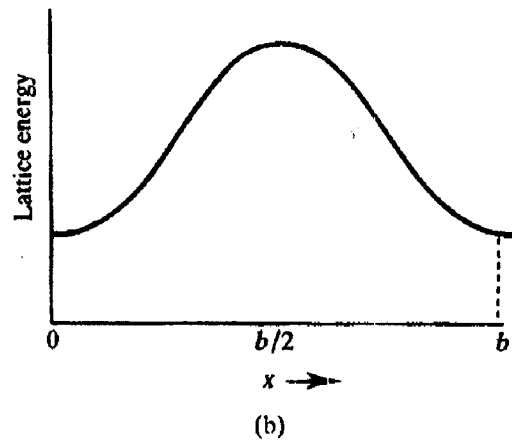
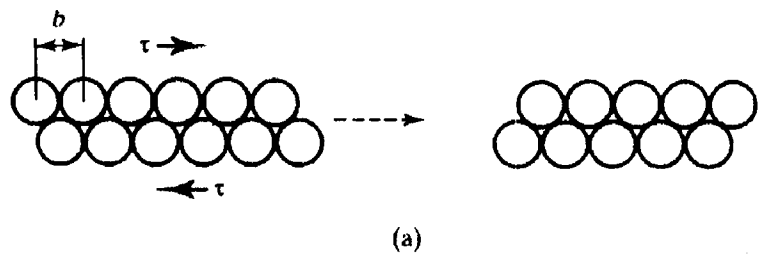


Figure 3.1

The process of slip in a perfect crystal. In (a), the shear stress acts to displace atoms in the upper row (plane) by the interatomic separation distance,  $b$ , with respect to the atoms in the lower plane. In (b) the system energy is plotted schematically vs. the displacement across the planes. Lattice energy is a maximum at the position  $x = b/2$ . In (c) the stress required to effect the displacement, proportional to the derivative of the energy-distance curve, is shown. The maximum stress is the theoretical shear strength. The solid curve is appropriate to the sinusoidal variation of energy with position shown in (b). The dotted line, which results in a lesser value of  $\tau_{\max}$ , more accurately represents the situation in a real material.

energy with relative atomic displacement across the rows is shown in Fig. 3.1*b*. When the atoms in the upper row have been displaced by one-half of their transit distance ( $b/2$ ), the crystal energy is at a maximum and atoms in the upper row are, in the absence of an applied stress, as likely to return to their initial position as to complete the slip transit. The internal restoring stress, which the applied stress must overcome, is related to the derivative of the energy-distance curve (cf. Chap. 2). The shear stress required to produce the atomic displacements described is indicated in Fig. 3.1*c*. The negative stress at displacements greater than  $b/2$  indicates that for such displacements the lattice force acts to complete the slip process. The applied stress required to overcome the lattice resistance to shear is indicated by  $\tau_{\max}$ ; it occurs at a displacement approximately equal to  $b/4$ . This stress represents the theoretical shear strength and it can be estimated by assuming that the stress-displacement curve varies in a sinusoidal manner, i.e.,

$$\tau = \tau_{\max} \sin\left(\frac{2\pi x}{b}\right) \quad (3.1)$$

At low (elastic) values of shear strain, Eq. (3.1) must satisfy  $\tau = G\gamma$  or, alternatively,  $d\tau/d\gamma = G$ . On differentiating Eq. (3.1),

$$\frac{d\tau}{dx} = \frac{2\pi}{b} \tau_{\max} \cos\left(\frac{2\pi x}{b}\right) \quad (3.2a)$$

and

$$\left(\frac{d\tau}{dx}\right)_{x=0} = \frac{2\pi}{b} \tau_{\max} \quad (3.2b)$$

For small displacements,  $\gamma = x/a$  (where  $a$  is the slip plane spacing) and on using  $d\tau/dx = (d\tau/d\gamma)(d\gamma/dx)$ , we obtain

$$\left(\frac{d\tau}{d\gamma}\right)_{x=0} = \frac{2\pi a}{b} \tau_{\max} \quad (3.3)$$

On setting Eq. (3.3) equal to  $G$ ,

$$\tau_{\max} = \frac{G}{2\pi} \frac{b}{a} \quad (3.4)$$

In Eq. (3.4),  $b$  (the slip distance) and  $a$  (the spacing between slip planes) are comparable. Thus, the theoretical shear strength is estimated to be on the order of  $G/2\pi$ . This estimate is somewhat high, since the asymmetry of the force-atomic separation curve is pronounced at the large strains occurring in the slip process (e.g.,  $\tan \gamma \cong 0.5$  at  $x = b/2$ ). A more realistic stress-displacement curve, which takes into account this asymmetry, is shown in Fig. 3.1*c*. Use of this more refined approach lowers the estimate of the theoretical strength to  $\tau_{\max} \cong G/30$ . Even this is much higher than shear yield strengths observed for single crystals of pure materials. For example,  $G/30$  for aluminum is ca.  $9 \times 10^8 \text{ N/m}^2$ , whereas the observed shear yield strength is  $7.8 \times 10^5 \text{ N/m}^2$ . Values of observed and theoretical shear strengths for other materials are compared in Table 3.1. In all cases the discrepancy between the two values is so great that there is no doubt that atomic shear, by which plastic deformation occurs in crystalline solids, does not take place by the process just described. Instead, it occurs by the movement of structural defects called dislocations,

Material	$\tau_{th} (= G/30) (10^9 \text{ N/m}^2)$	$\tau_{exp} (10^6 \text{ N/m}^2)$	$\tau_{exp}/\tau_{th}$	$\tau_f (10^6 \text{ N/m}^2)^*$
Ag	1.0	0.37	0.00037	20
Al	0.9	0.78	0.00087	30
Cu	1.4	0.49	0.00035	51
Ni	2.6	3.2	0.0070	121
$\alpha$ -Fe	2.6	27.5	0.011	150

\*Overestimate, based on narrow dislocation width, cf. Eq. (3.5).

Table 3.1

**Values of the theoretical, experimental, and frictional yield strengths for several materials**

the existence of which was postulated in the 1930s to explain the discrepancies between experimentally observed and theoretically calculated shear strengths. Further credence to the dislocation explanation of plastic deformation, as well as to the essential correctness of the theoretical shear strength calculations, was obtained when whiskers—dislocation-free crystals—were manufactured. The yield strengths of whiskers are much greater than those of ordinary materials, and within a factor of 10 of their theoretical shear strengths.

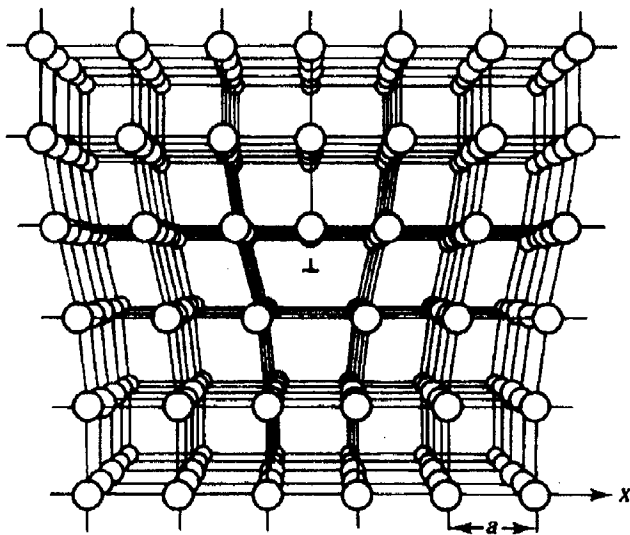
### 3.3 THE EDGE DISLOCATION

An appreciation of the atomic arrangements in the vicinity of the defects causing plastic flow is necessary to understand how they facilitate atomic slip. Thus, we begin our discussions on dislocations by describing these arrangements in the vicinity of a line defect called the edge dislocation, and then show how slip is caused by motion of this defect.

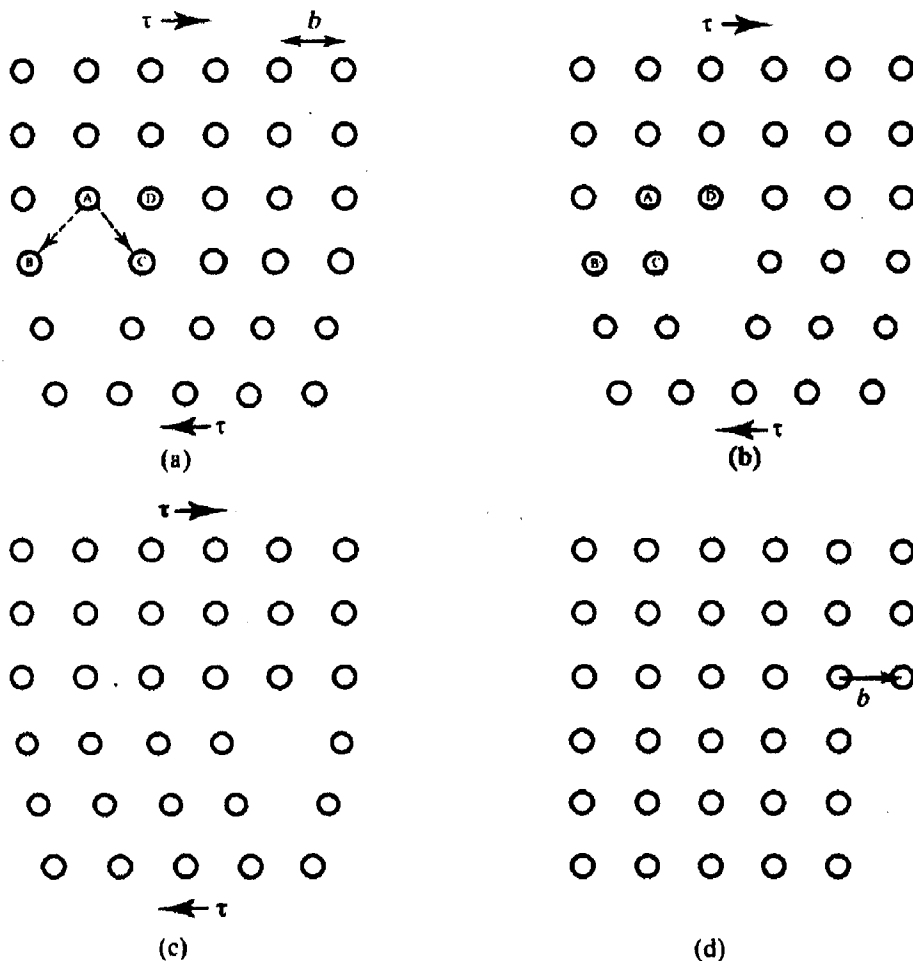
#### A. Slip by Edge Dislocation Motion

A sketch of an edge dislocation is shown in Fig. 3.2.<sup>1</sup> The defect can be considered an additional partial plane (commonly called an extra “half” plane) of atoms inserted into the upper portion of the crystal and terminating on a {100} plane. The means by which atomic displacements are achieved by edge dislocation motion are shown in Fig. 3.3. Here a shear stress is applied on the top and bottom faces of the crystal so as to produce shearing forces in a  $\langle 100 \rangle$  direction. Inspection of the atomic arrangement in the vicinity of the partial plane termination shows that, in the absence of an applied stress, the atom (atom A) at the termination is equally attracted to atoms B and C in the plane below it. Application of the stress alters this condition so that atom A is attracted preferentially to atom C. As a result of this,

<sup>1</sup>The simple cubic structure is used here to illustrate. Although the structure is a hypothetical one, it serves to describe the physics of dislocation structure and motion, and is used in preference to other structures for which the atomic arrangements in the vicinity of a dislocation are not so easily visualized.



**Figure 3.2**  
A schematic of an edge dislocation, represented by a partial atomic plane in a simple cubic structure. The "core" of the dislocation is localized at the partial plane termination. Atomic positions are distorted in the dislocation core region. (Adapted from A. G. Guy and J. J. Hren, *Elementary Physical Metallurgy*, 3rd ed., Addison-Wesley, Reading, Mass., 1974. Reprinted with permission.)



**Figure 3.3**  
Motion of an edge dislocation in response to a stress. (a) The stress biases atom A to lie closer to atom C than to B. (b) As a consequence the location of the core moves to atom D. (c) The process repeats and the dislocation moves to the right until (d) it exits the crystal effecting a displacement,  $b$ , of the top part of the crystal with respect to the bottom portion. (For clarity atoms in only one plane of the crystal are shown.)

minor atomic rearrangements occur in the vicinity of the dislocation core (i.e., in the region of the partial plane termination) to produce the arrangement shown in Fig. 3.3*b*. The new atomic positions are such that the dislocation has translated one atomic position to the right; that is, the extra half plane now terminates at atom D. We emphasize that the additional partial plane has not moved as such. Rather, the small displacements resulting from the applied shear stress relocate the position of the partial plane termination. With continued application of the stress, the dislocation continues to move in the direction indicated, and in the process atoms lying on the plane of the termination are displaced to the right relative to atoms on the plane below it. The plane containing the dislocation is called the slip plane, and the slip direction is the direction of the dislocation motion. The magnitude of the slip displacement is  $b$ , the interatomic separation in the slip plane, and represents the displacement of atoms on this plane relative to those on the plane below it. Continued motion of the dislocation from left to right (Fig. 3.3*c*), until it emerges from the side of the crystal (Fig. 3.3*d*), produces a permanent offset of magnitude  $b$  of the top part of the crystal with respect to the bottom one. That is, plastic deformation has resulted from the motion of the dislocation.

Comparison of the means by which permanent deformation is achieved by dislocation motion and by perfect slip is worthwhile. For the latter, every atom in the slip plane moves simultaneously with respect to atoms in the plane below it. This concurrent motion produces a displacement ( $b$ ) of the two portions of the crystal, and the force required for this is reflected in the theoretical shear strength. If slip occurs by edge-dislocation motion, the displacement  $b$  is effected only over one column of atoms at a time. It is only when the dislocation has moved the entire length of the slip plane that displacement of the two halves of the crystal by the distance  $b$  is achieved. Indeed, if the crystal is composed of  $N$  atomic columns, each time the dislocation moves a distance  $b$  in the slip direction, translation of the upper part of the crystal with respect to the lower part by an amount  $b/N$  takes place. It is this incremental nature of slip by dislocation motion that makes the stress required to produce it so much less than the theoretical strength associated with simultaneous slip of all atoms.

That the stress required to induce dislocation motion should be much less than the theoretical strength can be appreciated by closer inspection of Fig. 3.3. Indeed, intuition suggests that the dislocation motion stress should be vanishingly small, since atom A is attracted equally to atoms B and C and it would be expected that only a trivial "push" would be required to get the dislocation to "move over." However, the attractive and repulsive forces between atoms depend differently on interatomic separation (Chap. 2), and because of this the energy of the crystal increases somewhat as the partial plane termination moves from atom A to atom D. This variation in energy is illustrated schematically in Fig. 3.4 as is the stress, proportional to the derivative of the energy-distance curve, required to move the dislocation. However, this lattice *frictional stress* is much less than the theoretical strength. Peierls and Nabarro have calculated the frictional stress; it is given by

$$\tau_f = G \exp\left[\frac{-2\pi a}{(1-\nu)b}\right] \quad (3.5)$$

where  $G$  is the shear modulus,  $\nu$  Poisson's ratio,  $a$  the separation distance between slip planes, and  $b$  the slip distance. As given by Eq. (3.5), the frictional stress is

# 10

## Dislocations and Plastic Flow

### 10-1. INTRODUCTION; IMPERFECTIONS, IMPURITIES, AND PLASTIC FLOW

Imperfections in the over-all crystalline order and the presence of very small amounts of impurity atoms play a profound role in determining the observed plastic behavior of solids. The flow mechanism itself is based on the movement of dislocation type imperfections through the lattice since by this process the critical shear stress necessary for flow is drastically reduced. In addition, through interactions with dislocations that may impede or enhance their movement, other imperfections may also play a role of consequence. In fact, the rate of diffusion of point imperfections to or away from dislocation sites is usually the controlling factor that determines time-dependent inelastic flow behavior at high temperatures.

Imperfections are classified according to their dimensional extent. Thus, vacancies, Schottky and Frenkel defects, interstitialcies, and impurity atoms all fall into the category of *point defects*. Point defects do not disturb the lattice too much and, accordingly, have generally lower energies of formation. They are usually distributed in the lattice in arrangements that constitute stable-equilibrium configurations. On the other hand, *line imperfections*, such as dislocations, and *plane defects*, such as grain and twin boundaries, stacking faults, and external surfaces, represent relatively high-energy configurations. When present in the lattice these imperfections are found only in metastable equilibrium. As a result, the line and plane imperfections are generally more sensitive to changes in energy due to an applied stress.

In order to develop a theory of plastic flow, it is necessary to examine the geometry of dislocations and the stress interactions in which they participate. Furthermore, it is essential to consider structural mechanisms that give rise to certain characteristic types of inelastic behavior, and, in addition, to investigate how dislocations arise in the lattice.

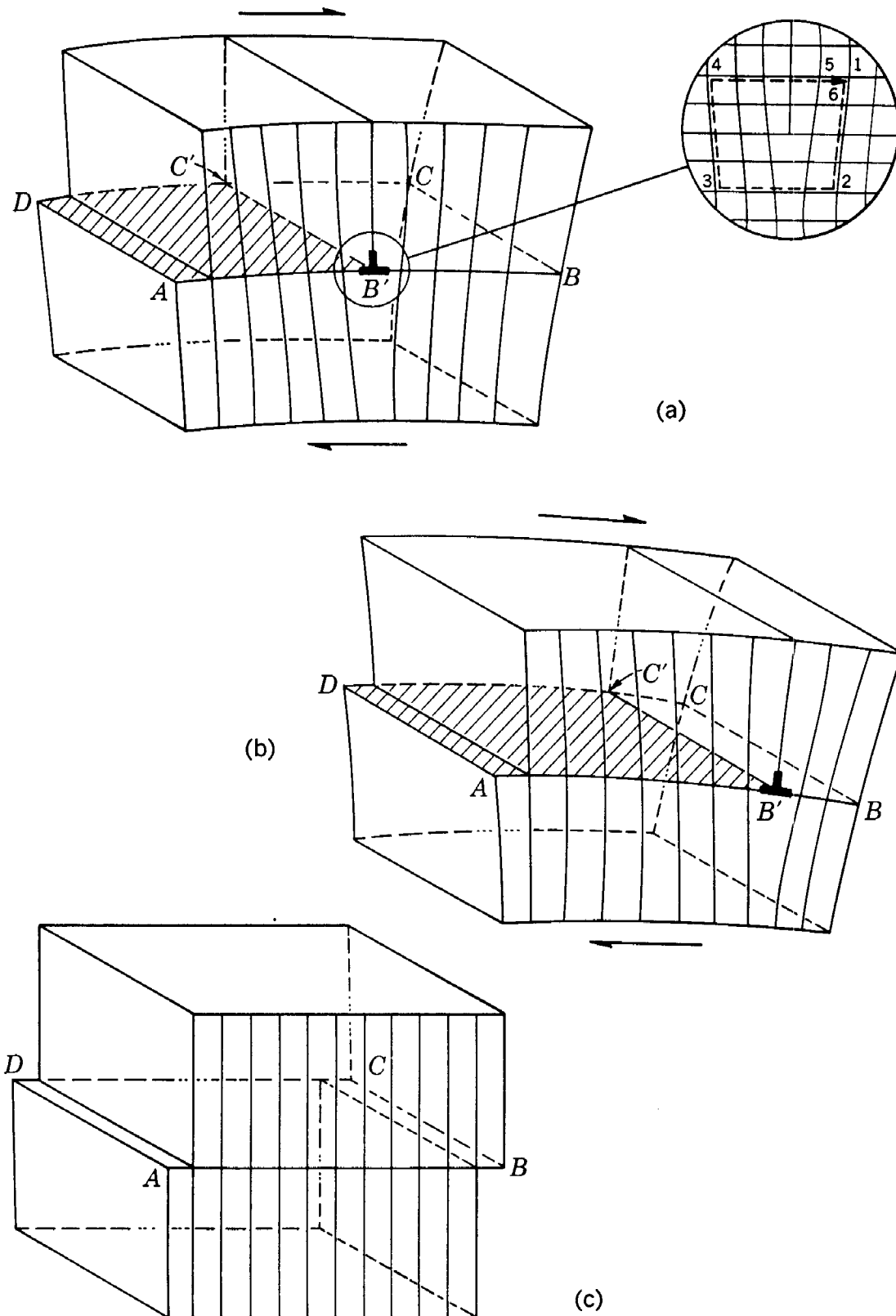


Fig. 10-1. Motion of an edge dislocation ( $B'C'$ ) in a simple cubic crystal. left to right (diagrams (a), (b), and (c)) is identical with that produced by a (e), and (f)). A Burgers

