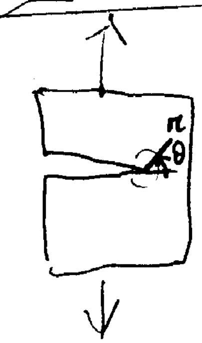
STRESSES AND DISPLACEMENTS NEAR A CRACK TIP

SHARP CRACK
PARALLEL FACES

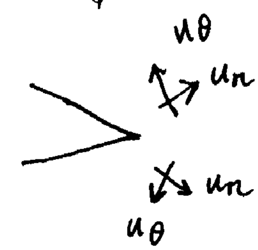
In-plane symmetric: Mode I

(SYMMETRIC)

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_{r\theta} \end{Bmatrix} = \left(\frac{1}{2\pi r}\right)^{\frac{1}{2}} K_I \cos\frac{\theta}{2} \begin{Bmatrix} \frac{1}{2}(3 - \cos\theta) \\ \frac{1}{2}(1 + \cos\theta) \\ \frac{1}{2}\sin\theta \end{Bmatrix}$$



$$\begin{Bmatrix} u_r \\ u_\theta \end{Bmatrix} = \left(\frac{r}{2\pi}\right)^{\frac{1}{2}} \frac{1}{\mu} K_I \begin{Bmatrix} \frac{1}{4}\left[(2\kappa-1)\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}\right] \\ -\frac{1}{4}\left[(2\kappa+1)\sin\frac{\theta}{2} - \sin\frac{3\theta}{2}\right] \end{Bmatrix}$$



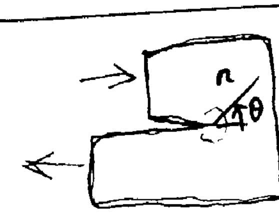
$u_r(r,\theta) = u_r(r,-\theta)$

$u_\theta(r,\theta) = -u_\theta(r,-\theta)$

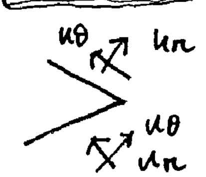
In-plane antisymmetric: Mode II

(ANTISYMMETRIC)

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_{r\theta} \end{Bmatrix} = \left(\frac{1}{2\pi r}\right)^{\frac{1}{2}} K_{II} \begin{Bmatrix} \frac{1}{2}(3\cos\theta - 1)\sin\frac{\theta}{2} \\ -\frac{1}{2}3\sin\theta\cos\frac{\theta}{2} \\ \frac{1}{2}(3\cos\theta - 1)\cos\frac{\theta}{2} \end{Bmatrix}$$



$$\begin{Bmatrix} u_r \\ u_\theta \end{Bmatrix} = \left(\frac{r}{2\pi}\right)^{\frac{1}{2}} \frac{1}{\mu} K_{II} \begin{Bmatrix} -\frac{1}{4}\left[(2\kappa-1)\sin\frac{\theta}{2} - 3\sin\frac{3\theta}{2}\right] \\ -\frac{1}{4}\left[(2\kappa+1)\cos\frac{\theta}{2} - 3\cos\frac{3\theta}{2}\right] \end{Bmatrix}$$



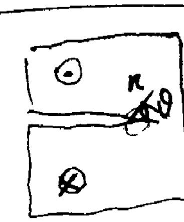
$u_r(r,\theta) = -u_r(r,-\theta)$

$u_\theta(r,\theta) = u_\theta(r,-\theta)$

Anti-plane: Mode III

(HOMEWORK)

$$\begin{Bmatrix} \sigma_{rz} \\ \sigma_{\theta z} \end{Bmatrix} = \left(\frac{1}{2\pi r}\right)^{\frac{1}{2}} K_{III} \begin{Bmatrix} \sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{Bmatrix}$$



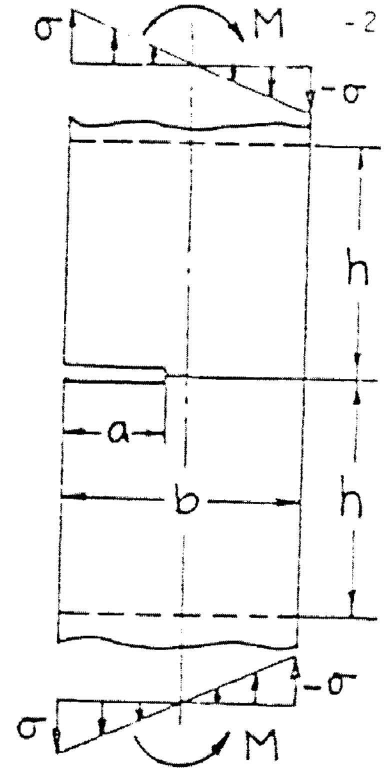
PLANE STRAIN $\kappa = 3 - 4\nu$

PLANE STRESS $\kappa = \frac{3-\nu}{1+\nu}$

$\mu = \frac{E}{2(1+\nu)}$

AN EXAMPLE FROM A COMPENDIUM OF STRESS INTENSITY FACTORS

THE PURE BENDING SPECIMEN



A. Stress Intensity Factor

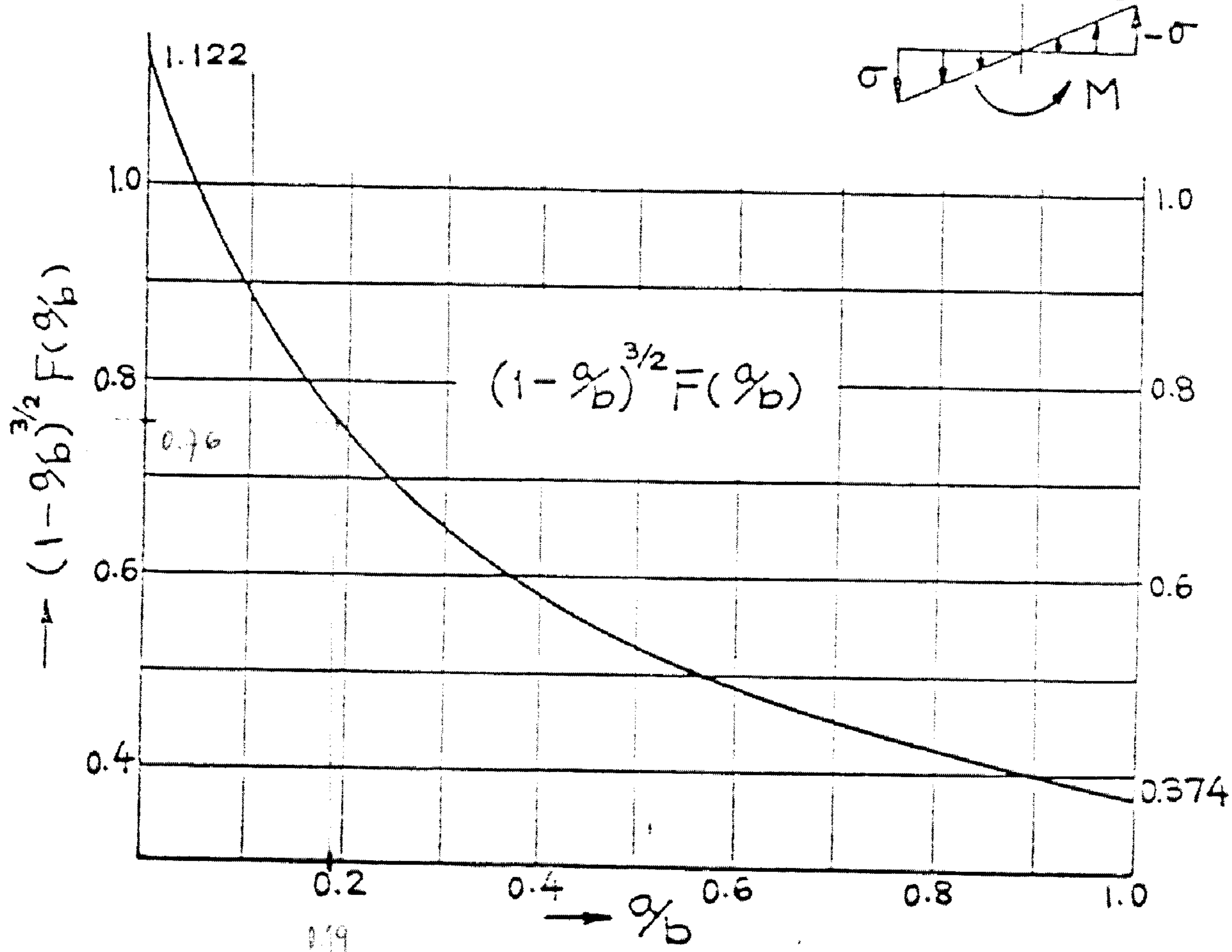
$$\sigma = \frac{6M}{b^2}$$

$$K_I = \sigma\sqrt{\pi a}\,F(a/b)$$

Numerical Values of F(a/b)

The curve in the following figure was drawn based on the results having better than 0.5% accuracy.

Used also: for 4 point bending.



Methods and References

1. Singular Integral Equation, Bueckner 1960
2. Boundary Collocation Method (h/b ≥ 2), Gross 1965
3. Weight Function Method, Bueckner 1970, 1971
4. Green's Function Method (h/b ≥ 1.5), Emery 1969
5. Asymptotic Approximation, Benthem 1972

ELASTIC FIELDS FOR STUDY OF GEOMETRIC SINGULARITIES

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		$\varphi(z)$	$\psi(z)$	U	$2\mu u_r$	$2\mu u_\theta$
symmetric	A	$z^{1+\lambda}$	0	$r^{2+\lambda}\cos\lambda\theta$	$(\kappa-1-\lambda)r^{1+\lambda}\cos\lambda\theta$	$(\kappa+1+\lambda)r^{1+\lambda}\sin\lambda\theta$
	B	0	$z^{1+\lambda}$	$\frac{1}{2+\lambda}r^{2+\lambda}\cos(2+\lambda)\theta$	$-r^{1+\lambda}\cos(2+\lambda)\theta$	$r^{1+\lambda}\sin(2+\lambda)\theta$
anti-sym	C	$iz^{1+\lambda}$	0	$-r^{2+\lambda}\sin\lambda\theta$	$-(\kappa-1-\lambda)r^{1+\lambda}\sin\lambda\theta$	$(\kappa+1+\lambda)r^{1+\lambda}\cos\lambda\theta$
	D	0	$iz^{1+\lambda}$	$-\frac{1}{2+\lambda}r^{2+\lambda}\sin(2+\lambda)\theta$	$r^{1+\lambda}\sin(2+\lambda)\theta$	$r^{1+\lambda}\cos(2+\lambda)\theta$

	σ_{rr}	$\sigma_{r\theta}$	$\sigma_{\theta\theta}$
A	$(1+\lambda)(2-\lambda)r^{\lambda}\cos\lambda\theta$	$\lambda(1+\lambda)r^{\lambda}\sin\lambda\theta$	$(1+\lambda)(2+\lambda)r^{\lambda}\cos\lambda\theta$
B	$-(1+\lambda)r^{\lambda}\cos(2+\lambda)\theta$	$(1+\lambda)r^{\lambda}\sin(2+\lambda)\theta$	$(1+\lambda)r^{\lambda}\cos(2+\lambda)\theta$
C	$-(1+\lambda)(2-\lambda)r^{\lambda}\sin\lambda\theta$	$\lambda(1+\lambda)r^{\lambda}\cos\lambda\theta$	$-(1+\lambda)(2+\lambda)r^{\lambda}\sin\lambda\theta$
D	$(1+\lambda)r^{\lambda}\sin(2+\lambda)\theta$	$(1+\lambda)r^{\lambda}\cos(2+\lambda)\theta$	$-(1+\lambda)r^{\lambda}\sin(2+\lambda)\theta$