

Moon Patterns, Sun Patterns, and Wave Breaking in Rotating Granular Mixtures

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Granular materials, such as powders and sand, tend to segregate due to differences in particle properties. When a cylindrical drum is partially filled with particles of different sizes and rotated about its axis, this leads to radial segregation patterns in which the smaller particles concentrate in a radial core near the axis, and the larger particles near the outside walls of the drum. Under certain conditions, undulations in the radial core of smaller particles grow into radial stripes that extend toward the outer walls of the drum in a manner somewhat reminiscent of viscous fingering. The patterns are strongly dependent on the fill level and rotation speed of the drum. These observations can be explained by two spatially disjoint mechanisms: (1) a wave-breaking mechanism that promotes the growth of the stripes and (2) a filtering mechanism that limits the growth of stripes.

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INTRODUCTION

When a granular mixture is vibrated, sheared, or otherwise disturbed, it segregates by type of grain [1–5]. For instance, if a bed of grains of different sizes is sheared parallel to the top surface, the larger grains segregate to the surface of the bed and the smaller grains to the bottom [6]. In nature, this form of granular segregation might explain why the gravelly bed of a piedmont stream is frequently topped with a self-organized, stabilizing *armour layer* composed of the largest rocks in the bed [7]. Granular segregation also occurs in numerous industrial processes, often with undesirable effects [8]. For example, segregation is a problem in the pharmaceu-

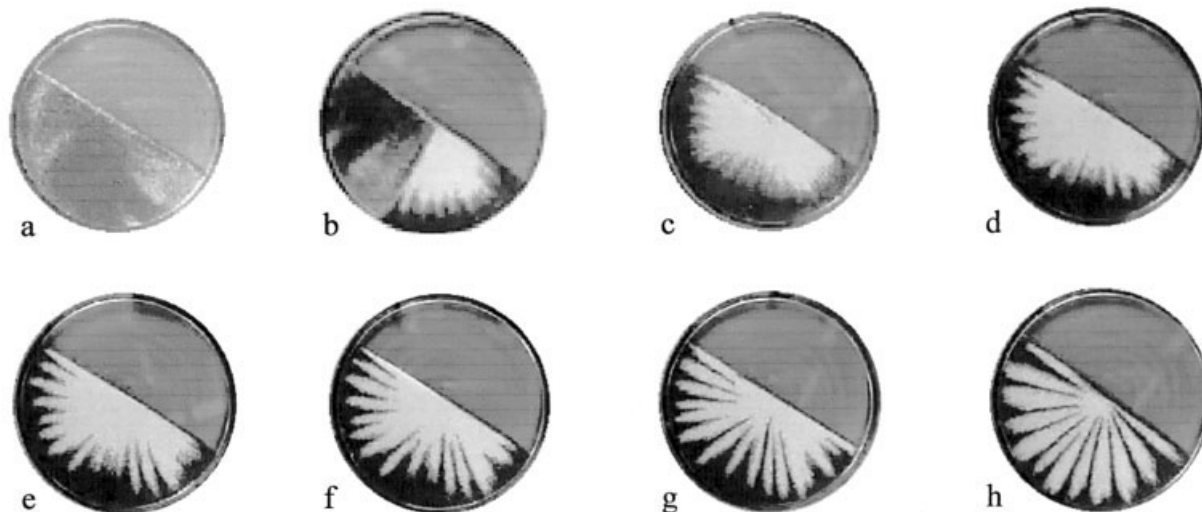
tical industry, where the components of pills must be well mixed to ensure the uniform distribution of medication.

To study granular segregation, some researchers have used a thin rotating drum partially filled with a mixture of beads of two different sizes. For example, consider the nearly half-filled drum in Figure 1. Within the first rotation of the drum, a semi-circular radial segregation pattern forms [Figure 1(a–c); 9–12]. We call the pattern of Figure 1(c) a *moon pattern* because it reminds us of an ashy moon on a dark sky. The moon pattern forms very quickly and then evolves through a sequence of increasingly wavy striped segregation patterns—or *sun patterns*—as the rotation continues [Figure 1(d–g); 13, 14]. After about 10 rotations, the evolution stops, and the sun pattern remains steady thereafter [Figure 1(h); 13, 14].

In this article we review what is known about the development of moon patterns. Then we show that, because of the

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FIGURE 1



This shows the segregation patterns that form in a rotating drum partially filled with 0.8 mm (white) and 2.0 mm (black) glass beads. The interstitial fluid is water. (a) Photograph of the drum in the initial state: the beads are initially well mixed. (b) Photograph of the drum after a quarter rotation showing a partially formed moon pattern. (c) Photograph of the drum after one half rotation showing a complete moon pattern. (d–g) Photographs of the drum after two, three, four, and five rotations, showing the evolution of the stripes. (h) A photograph of the drum after 10 rotations, showing a steady sun pattern.

inevitable imperfections in the geometry of any moon pattern and by virtue of a wave-breaking mechanism, moon patterns tend to become unstable and subsequently to evolve into sun patterns. We also study experimentally and discuss theoretically how the sun patterns vary with the angular velocity of the drum. Last, we show experimentally that, although the moon patterns are always unstable and tend to form striped patterns, this tendency can only manifest itself in a narrow range of fill levels of the drum, close to one half full. When the fill level is outside that narrow range, the moon pattern appears to be steady. We ascribe these experimental results to a fill-level dependent filtering mechanism that restricts the amplitude-to-wavelength ratio of the sun patterns.

EXPERIMENTAL SET-UP

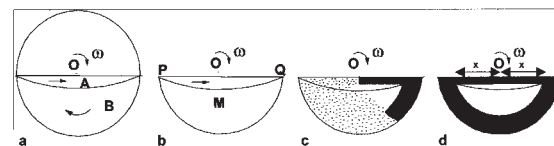
In our experiments we use a thin, circular drum of diameter ~ 300 mm and thickness ~ 6.4 mm (perpendicular to the pictures in Figure 1) partially filled with a mixture of glass beads of two different sizes. The mixture consists of black beads of diameter 2 mm (40% by weight) and white beads of diameter 0.8 mm (60% by weight). The interstitial fluid is water, though similar results are seen if air is in the interstitial fluid. Using water as the interstitial fluid eliminates problems resulting from static electricity. Throughout the experiments, we rotate the drum at a constant angular velocity ω in the range $0.45 \text{ rpm} < \omega < 5.4 \text{ rpm}$. In that range the flow of the granular materials is steady and the free surface of the granular materials remains flat. We control the angular velocity with a stepper motor (Compumotor)

and take digital pictures at regular intervals in time during each experiment using a digital camera (Cohu).

FORMATION OF MOON PATTERNS

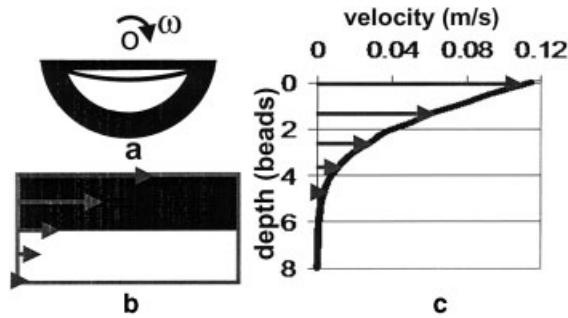
When a granular material is rotated slowly in a drum, the particles flow (i.e., they move relative to each other) only in a thin surficial layer—the *flowing layer* [marked A in Figure 2(a)]. Outside the flowing layer, in the region marked B in Figure 2(a), the beads move in solid-like rotation with the drum. The flowing layer exchanges beads with the underlying substratum through the shallow arc PMQ of Figure 2(b). The beads positioned along the arc PM can be said to be *thawing* as they enter the flowing layer and cease to move in solid-like rotation with the drum. Thus we can call PM the *thawing arc*. Similarly, the beads positioned along the arc

FIGURE 2



(a) Sketch of the drum. (The free surface is shown horizontal.) "A" is the thin flowing layer. "B" is the frozen substratum and ω is the angular velocity. (b) Illustration of the freezing and thawing arcs (see text). (c) and (d) Cartoon of the formation of a moon pattern (see text).

FIGURE 3



(a) Illustration of an idealized moon pattern. (b) Detail of the small portion of the flowing layer boxed in 4(a) showing the horizontal interface between small and large beads. The arrows indicate the velocity at five different depths within the flowing layer. (c) Measured velocity field.

MQ can be said to be *freezing* as they exit the flowing layer and begin again to move in solid-like rotation with the drum. Thus we can call MQ the *freezing arc*.

Consider now the cartoon of Figure 2(b–d). Because the black beads and white beads are initially well mixed, they enter the flowing layer through the thawing arc in a flux of constant composition [i.e., in a flux whose average composition is constant in time; Figure 2(b)]. As they move in the flowing layer, the larger, black beads segregate to the upper part of the layer and the smaller, white beads segregate to the lower part. Later, when these beads start to leave the flowing layer through the freezing arc, they begin to form a moon pattern [Figure 2(c)]. Last, when the beads frozen in the partially formed moon pattern reenter the flowing layer (again in a flux of constant composition), the moon pattern becomes complete [Figure 2(d)]. In experiments, moon patterns are observed to form just as in Figure 2(b–d), that is to say, within one rotation of the drum and for all fill levels (see, for example, Refs. 9–12). Figure 2(b, c, and d) show experimental results which correspond roughly to Figure 1(a, b, and c), respectively.

INSTABILITY OF MOON PATTERNS: THE WAVE-BREAKING MECHANISM

Consider the idealized moon pattern of Figure 2(d). In this pattern, the flux of beads entering the flowing layer through the thawing arc is of constant composition. That is to say, the rate at which both the small and the large beads enter the flowing layer is steady. Under these idealized conditions, the moon pattern may persist in time as the drum rotates. To understand why, let us focus on the small portion of the flowing layer boxed in Figure 3(a). The flowing layer is segregated, with the larger beads concentrated at the top and the smaller beads concentrated at the bottom [Figure 3(b)]. Because the flux of beads entering the flowing layer is of constant

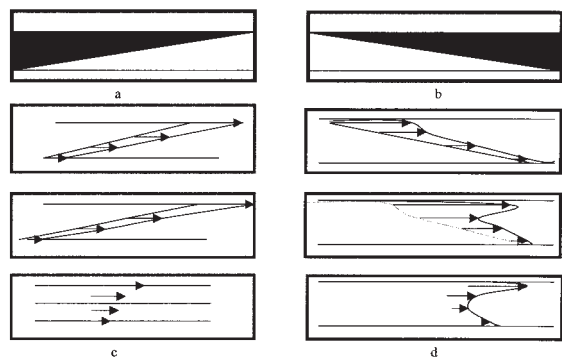
composition, the interface between the larger beads and the smaller beads is parallel to the free surface. The velocity in the flowing layer is also parallel to the free surface and decreases from top to bottom as sketched in Figure 3(b). [The measured velocity field is shown in Figure 3(c).] Therefore, the velocity at the interface is parallel to the interface itself; the interface is not affected by the flow and remains invariant under the idealized conditions of Figure 3(a).

The photograph in Figure 1(c) indicates that in actuality the moon pattern is not perfectly semicircular as in Figure 1(d), but instead it is slightly wavy. Because of this slight waviness, the flux of beads entering the flowing layer through the thawing arc is no longer of constant composition. Instead, the percentage of large beads entering the flowing layer increases and decreases periodically in time (and the same is true of the percentage of small beads). Therefore, the interface between the small beads and the large beads does not remain parallel to the free surface. Rather, the slope of the interface alternates between a positive value when the percentage of large beads is increasing in time [Figure 4(a)] and a negative value when the percentage of large beads is decreasing in time [Figure 4(b)]. Because the velocity is always parallel to the free surface, it is clear in both Figure 4(a) and Figure 4(b) that, regardless of the sign of its slope, the interface is not parallel to the velocity. Furthermore, because the velocity changes with depth, the interface may be advected, resulting in an evolving interface.

We now discuss how the interface evolves when the interface is not parallel to the free surface. The upper portion of the interface is always advected faster than the lower portion. When the slope of the interface is initially positive, the slope becomes less and less steep as the interface is advected by the velocity field, as sketched in Figure 4(c). When the slope of the interface is initially negative, the upper portion of the interface overtakes the lower portion and is projected forward in the form of an incipient stripe. We term this phenomenon *wave breaking*. Experimental evidence of wave-breaking is apparent in the sequence of photographs of Figure 5.

The preceding discussion indicates that the stripes originate in wave-breaking events within the flowing layer and, therefore, that the thickness of the stripes is bound by the thickness of the flowing layer. Because the thickness of the flowing layer increases with the angular velocity of the drum [15–17], it follows that the thickness of the stripes should increase (and therefore the number of stripes decrease) as a result of an increase in the angular velocity. This trend is indeed confirmed by the experimental results shown in the first column of Figure 6. The photographs in this column show the steady sun patterns for five angular velocities at a single fill level of about 50%. The same trend is observed at other fill levels close to 50%, as shown in the second and third columns of Figure 6, though the steady sun pattern varies with fill level at any given speed. This brings us to the influence of the fill

FIGURE 4



Cartoons illustrating how the interface between large (black) and small (white) beads is advected by the velocity field when the slope of the interface is positive (a) and negative (b). A positive slope corresponds to an increasing percentage of large beads in time, whereas a negative slope corresponds to a decreasing percentage of large beads in time. The interface with a positive slope is advected down to a horizontal position (c) whereas the interface with a negative slope undergoes *wave breaking* (d), as described in text.

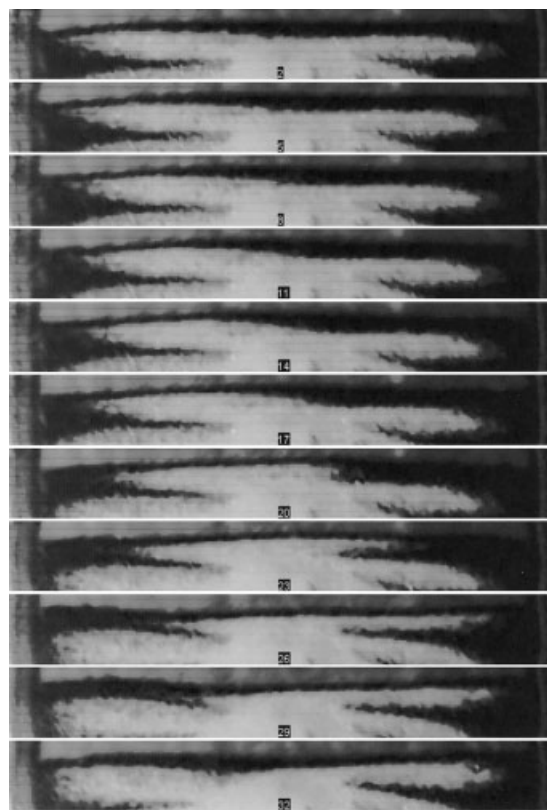
level on the steady sun pattern, which we discuss in the next section.

EFFECT OF FILL-LEVEL: THE LOW-PASS FILTERING MECHANISM

Figure 7 summarizes our experimental observations on how the evolution of radial segregation patterns depends on the fill level of the drum. (For all images in the figure, $\omega = 1$ rpm.) The five pictures in each row in Figure 7 show the evolution of the segregation patterns for fixed fill levels of the drum. For the range of fill levels between roughly 50 and 61%, the segregation patterns evolve as in the experiment of Figure 1: in each case, a moon pattern forms quickly, becomes unstable, and evolves through a sequence of increasingly wavy sun patterns as the drum continues to rotate. However, for fill levels of less than 50% and for fill levels of more than 61%, *the moon patterns are stable*. This appears to contradict our conclusion that, by virtue of the wave-breaking mechanism, moon patterns tend to become unstable and subsequently to evolve into sun patterns.

To dispel this apparent contradiction, consider Figure 8(a and b). In Figure 8(a), O is the axis of the drum, and $|d'|$ is the distance from O to the bottom of the flowing layer. For the sake of simplicity, we substitute the arc PMQ of Figure 2(b) (which represents the bottom of the flowing layer) with the straight line LMR of Figure 8(a) (which is parallel to the free surface). Then, the line LM is an idealized thawing arc, and the line MR is an idealized freezing arc. The beads positioned along the freezing line MR at a given time remain on a straight line after moving for a short while in solid-like rotation with the drum [Figure 8(b)]. Figure 8(c) shows the

FIGURE 5



A series of photographs showing the wave breaking of an interface between large and small beads. The numbers refer to frames in a video taken at 30 frames per second. The free surface is visible at the top of each photograph. The upper half of the photograph corresponds approximately to the flowing layer. The dark beads are large (diameter ~ 2 mm) and the white beads are small (diameter ~ 0.8 mm).

position of these same beads at regular intervals of time as they continue to rotate with the drum.

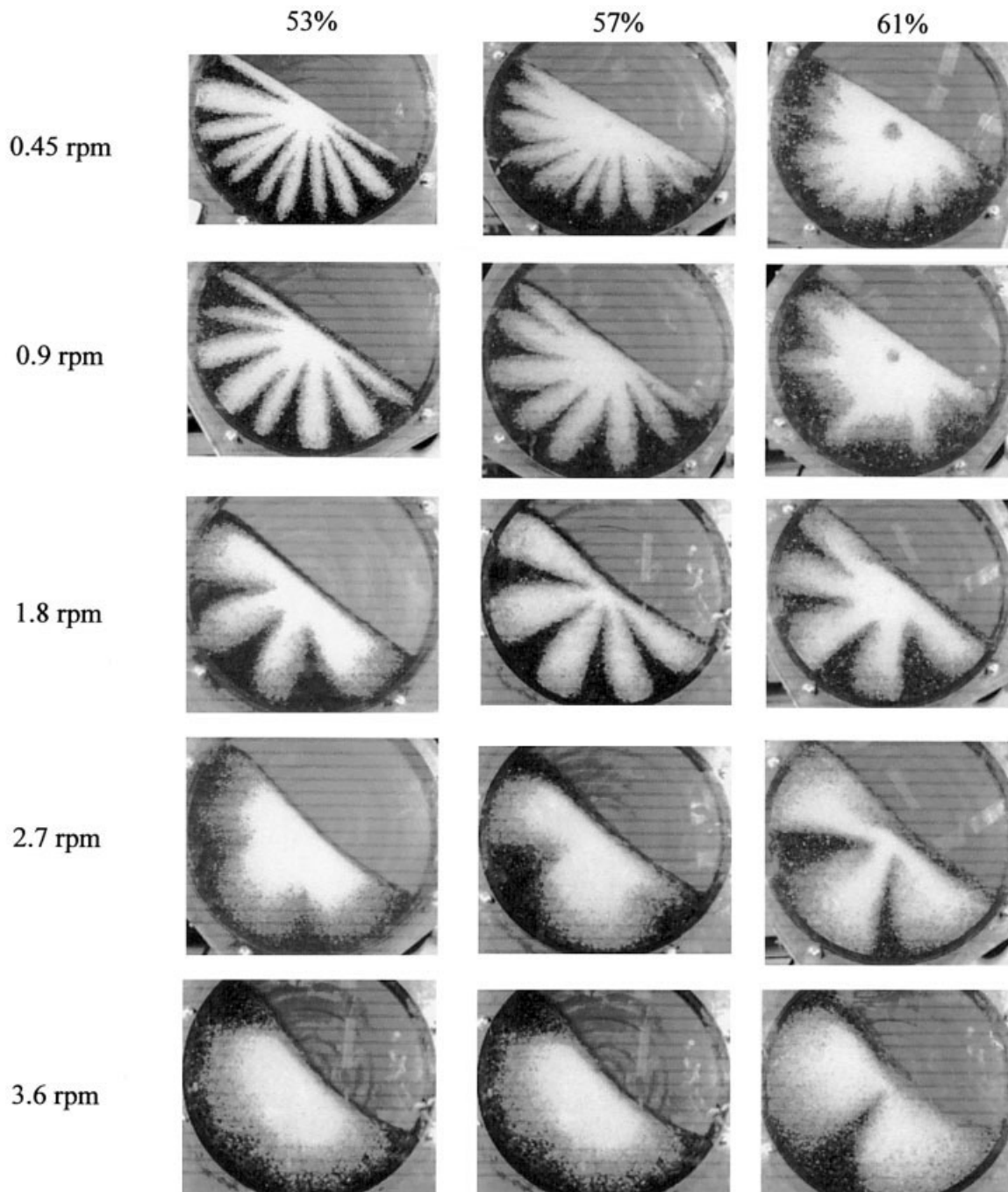
In this context, let us consider the sequence in which the beads leave the flowing layer in an idealized sun pattern.

For a time interval $T/2$, the beads that leave the flowing layer are black; then, for a time interval $T/2$, the beads that leave the flowing layer are white, and so on in a cyclic fashion. Thus the beads leave the flowing layer in a flux of oscillating composition (a black stripe, then a white stripe, and so on). The ensuing stripes unfold in the fanlike arrangement of Figure 8(c).

Now let us consider the sequence in which the beads reenter the flowing layer in the same idealized sun pattern.

If $|d'|$ is small compared to the radius of the drum (i.e., if the fill level is close to 50%), then the beads reenter the flowing layer in a flux of oscillating composition (a black stripe, then a white stripe, and so on). In this case, the

FIGURE 6






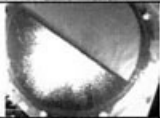

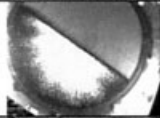




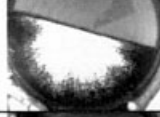


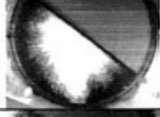












































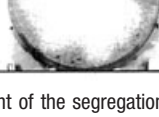

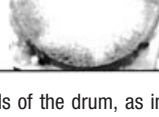
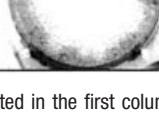
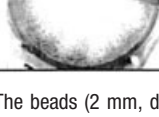


The pictures in this chart show how the steady striped segregation patterns vary with the angular velocity. The first column in this chart shows how the steady striped radial segregation patterns vary with angular velocity for a fill level of about 50%: The faster the rotation speed, the fatter the stripes. The second and third columns show steady segregation patterns for fill levels of 57 and 61%, respectively, indicating that this trend is independent of fill level.

compositional oscillations of the flux that leaves the flowing layer carry over to the flux that reenters the flowing layer, and the sun pattern may persist.

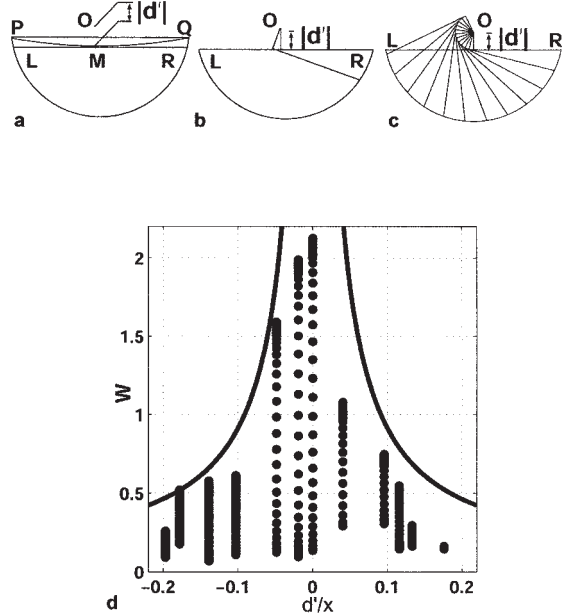
On the other hand, if $|d'|$ is sizeable fraction of the radius of the drum (i.e., if the fill level is either substantially lower or higher than 50%), then the beads leave the flowing layer

FIGURE 7

Fill Level	5 Rotations	15 Rotations	25 Rotations	35 Rotations	45 Rotations
48%					
49%					
50%					
51%					
52%					
53%					
54%					
55%					
57%					
58%					
61%					
63%					
65%					

Pictures of the development of the segregation pattern for different fill levels of the drum, as indicated in the first column. The beads (2 mm, dark, and 0.8 mm, white) were initially well mixed. We took pictures at regular intervals in time as the drum rotated at a constant angular speed of 1 rpm. The time of the experiment is noted in the first row in units of mixture rotations.

FIGURE 8



(a) PQ is the free surface; the arc PMQ is the bottom of the flowing layer; PM is the thawing arc; MQ is the freezing arc; LM is the thawing line, and MR is the freezing line. Beads that freeze at the same time remain on a straight line (b) as they rotate with the drum (c). (d) A comparison of the predictions of the model with the experimental results. W is the waviness, and d'/x is an index of the fill level. The curve represents the predicted cutoff waviness. Each point represents an experimental measurement. Points with the same d'/x correspond to the same experiment; successive points with the same d'/x draws near the cutoff waviness as the pattern in the experiment becomes steady. For the definition of x , see Figure 2(d).

in a flux of constant composition. In this case, the compositional oscillations of the flux that leaves the flowing layer do not carry over to the flux that reenters the flowing layer, and the sun pattern may not persist.

In accord with these qualitative conclusions, a detailed analysis of the fluxes [13] that leave and reenter the flowing layer shows that the waviness of a sun pattern (i.e., the amplitude-to-wavelength ratio of its stripes) is subjected to low-pass filtering with a cutoff waviness that depends strongly on the fill level of the drum. Figure 8(c) shows a comparison of this prediction with experimental measurements from the experiments of Figure 7. The effect of the low-pass filtering is manifest in Figure 8(c): The experimental points lie beneath the theoretical curve showing the cutoff waviness of the filter. Further, for any given value of d'/x the waviness of a transient pattern draws near the cutoff waviness as the pattern becomes steady, indicating that the waviness of a steady pattern is *selected* by the low-pass filtering [13].

In summary, the fill-level dependence of the sun patterns can be explained as follows. By virtue of the wave-breaking mechanism, moon patterns are unstable, regardless of the fill level of the drum. If the fill level is close to 50%, the cutoff waviness is high, and the instability of the moon pattern may manifest itself. In Figure 7, this is the case for fill levels between 50 and 61%. On the other hand, if the fill level is substantially lower or higher than 50%, the cutoff waviness is very small, and, therefore, only sun patterns of very low waviness (essentially moon patterns) may exist, and the moon patterns appear to be stable. In Figure 7, this is the case for fill levels less than 50% or greater than 61%.

DISCUSSION AND SUMMARY

We have studied experimentally the radial segregation patterns that form in a rotating drum partially filled with a granular mixture. Within one rotation of the drum, a simple semi-circular segregation pattern that we have termed the “moon pattern” forms for all fill levels of the drum. Upon becoming unstable, moon patterns tend to evolve into striped patterns that we have termed “sun patterns.”

To gain some insight into how the moon patterns become unstable, we have focused on the kinematics of the flowing layer. The flowing layer is segregated parallel to the free surface, with the large beads in the upper part and the small beads in the lower part. By studying how the interface between the small beads and the large beads is advected by the velocity field in the flowing layer, we concluded that the initiation of stripes may be ascribed to a wave-breaking mechanism whereby the upper portion of the interface overtakes the lower portion to rush forward and freeze as an incipient stripe upon leaving the boundary layer. This mechanism appears related to a wavy instability reported by Goldfarb et al. [18]. We have documented experimentally one of these wave-breaking events in a series of photographs.

Last, we have established that, by virtue of a low-pass filtering mechanism that operates outside the flowing layer, the instability of the moon pattern only manifests itself for fill levels near 50%. We have also established that this low-pass filtering mechanism selects the waviness of sun patterns. Thus, we have found that, in contrast to many systems in which a wavelength is selected at the onset of a runaway process (usually embodied by an eigenproblem), in sun patterns the waviness (or amplitude-to-wavelength ratio) is selected when a low-pass filter arrests a runaway process of waviness amplification.

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