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Towards a theoretical model of localized turbulent scouring

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ABSTRACT: We examine theoretically the dynamic equilibrium between a jet-driven turbulent cauldron and a cohesionless granular bed. The turbulent cauldron forms in a pool of water whose depth is initially constant; driven by the jet, the turbulent cauldron scours a pothole in the granular bed. First, we use dimensional analysis and similarity methods to derive a theoretical formula for the equilibrium depth of the pothole. Whereas the empirical formulas customarily used in applications contain numerous free exponents, the theoretical formula contains a single similarity exponent, whose value remains undetermined. Second, we show that the same formula as well as the value of the similarity exponent can be obtained via the phenomenological theory of turbulence. Third, we show that our predictions compare well with experimental results. An unprecedented aspect of our theoretical analyses is that we focus on the energetics of the turbulent cauldron. Focusing on the energetics of the turbulent cauldron may also prove useful in developing a theoretical understanding bridge pier-induced erosion, mine burial, and other applications in which a localized turbulent flow interacts with a granular bed.

1 INTRODUCTION

Numerous applications in hydrology, geomorphology, and hydraulic engineering involve a water jet that plunges into a pool of water with a granular bed for a bottom. Then, a turbulent cauldron forms in the pool of water and starts to scour a pothole there (Figure 1). Under a sustained action of the jet, the pothole deepens until a state of dynamic equilibrium is attained between the granular bed and the turbulent cauldron. For example, in the bed of a stream below an overflowing gate, a pothole forms and may compromise the stability of the gate (Graf 1998). In this and other applications, it is necessary to estimate the equilibrium depth of the pothole. Over the years, researchers have proposed a number of widely used empirical formulas for the depth of the pothole (Mason & Arumugam 1985, Hoffmans & Verheij 1997). These formulas have been predicated on dimensional analysis and heuristic arguments, and contain numerous free exponents that have been determined by fitting experimental results. (In a few cases, one or more exponents have been determined theoretically, however, see, for example, Bormann & Julien (1991)). A review of the available empirical formulas leads to the following observations: (1) The formulas often lack dimensional homogeneity; (2) The formulas have often been the product of mangled attempts at dimensional analysis; (3) The formulas have often been predicated on limited experimental data; (4) The formulas have sometimes disregarded important parameters such as the diameter of the grains of the bed; (5) The formulas do not provide much insight into the interaction between the granular bed and the turbulent cauldron. Yet, for lack of better means, we continue to use empirical formulas to deal with many common applications of

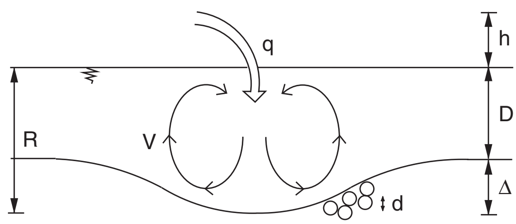

Figure 1. Geometry and notation. The jet of volume flux q plunges from a height h (called the *head*). The turbulent cauldron is spanned by its largest eddies (of characteristic velocity V). The granular bed is composed of cohesionless grains of diameter d. Note that the geometry is two-dimensional: the jet, the turbulent cauldron, and the pothole extend to infinity in the direction perpendicular to the plane of the figure (or thickness).

turbulent flows on which a century of progress in the theory of turbulence appears to have been largely inconsequential.

In this paper, we use dimensional analysis and similarity methods (Barenblatt 1996) to derive a theoretical formula containing a single free exponent—a similarity exponent. This formula subsumes the empirical formulas proposed so far and is valid asymptotically under conditions that are amply met in applications. Then we show that the same theoretical formula as well as the value of the similarity exponent can be derived using the phenomenological theory of turbulence (Frisch 1998). To that end, we build on recent work (Knight & Sirovich 1990, Lundgren 2002, Lundgren 2003) indicating that the phenomenological theory may be applied to turbulence that is both anisotropic and inhomogeneous (as is the case in the turbulent cauldron).

2 REVIEW OF EMPIRICAL AND SEMI-EMPIRICAL FORMULAS

Exhaustive compilations of empirical formulas for the equilibrium depth of the pothole were undertaken by Mason & Arumugam (1985), Breusers & Raudkivi (1991), and Hoffmans & Verheij (1997). From those compilations, it is apparent that all empirical formulas proposed so far are but special cases of the following generalized empirical formula:

$$R = K q^{e_q} h^{e_h} g^{e_g} d^{e_d} \left(\frac{\rho}{\rho_s - \rho} \right)^{e_\rho}, \qquad (1)$$

where R is the sum of the depth of the pothole and the depth of the pool of water, $R = \Delta + D$, and coincides with the size of the turbulent cauldron (Figure 1); K is a multiplicative constant whose value must be determined empirically; q is the volume flux of the jet per unit thickness (measured in the direction perpendicular to the plane in Figure 1); h is the head of the jet; g is the gravitational acceleration; d is the diameter of the grains of the bed; ρ is the density of water; ρ_s is the density of the grains of the bed; and e_q, e_h, e_g, e_d, and e_ρ are exponents whose values must be determined empirically. The rows of Table 1 list the values of the exponents determined empirically (or set to zero from the start) by different researchers. (In the case of Bormann & Julien, the values of the exponents were determined semi-empirically (Bormann & Julien 1991)). Each researcher (or group of researchers) determined the values of several free parameters (including a number of exponents and the multiplicative constant K) by fitting different experimental results. As might have been surmised from the number of free parameters and the diversity of experimental results, and Table 1 confirms, in some cases different researchers obtained widely dissimilar values of a given exponent.

Table 1. Sets of values of the exponents of Equation (1) empirically determined (or set to zero) by different researchers. Adapted from Mason & Arumugam (1985) and Hoffmans & Verheij (1997). Here "Eggenberger" refers to Eggenberger & Muller (1944), "Chee (A)" to Chee & Padiyar (1969), "Chee (B)" to Chee & Kung (1974), and "Bormann" to Bormann & Julien (1991). Also shown are the theoretical values of the exponents determined here.

Res.(s)	e_q	e_h	e_g	e_d	e_ρ
Schoklitsch	0.57	0.2	0	−0.32	0
Veronese	0.54	0.225	0	−0.42	0
Eggenberger	0.6	0.5	−0.3	−0.4	4/9
Hartung	0.64	0.36	0	−0.32	0
Franke	0.67	0.5	0	−0.5	0
Kotoulas	0.7	0.35	−0.35	−0.4	0
Chee (A)	0.67	0.18	0	−0.063	0
Chee (B)	0.6	0.2	0	−0.1	0
Bormann	0.6	0.5	−0.3	−0.4	0.8
SOFRELEC	0.6	0.1	0	0	0
INCYTH	0.5	0.25	0	0	0
Theory	2/3	2/3	−1/3	−2/3	1

3 DIMENSIONAL ANALYSIS AND SIMILARITY

In this section, we ascertain to what extent a theoretical formula may be predicated on dimensional analysis and similarity methods.

3.1 *Dimensional analysis*

We start by choosing a suitable set of variables. Because the turbulence is fully developed in applications, the viscosity need not be included in our set of variables. After evaluating a number of alternatives, we decide for the following set of 6 variables: R, ρ, g, ρ_s, d, and P. P is the power of the jet per unit thickness, $P = q\rho g h$, and therefore the power that sustains the turbulent cauldron. The dimensional equations $[P] = [\rho][g]^{3/2}[R]^{5/2}$, $[\rho_s] = [\rho]$, and $[d] = [R]$ show that the dimensions of three of the variables (P, ρ_s, and d) can be expressed as products of powers of the dimensions of the other variables; it follows from Buckingham's Π theorem (Barenblatt 1996) that we can reduce the functional relation among P, R, ρ, g, ρ_s, and d to an equivalent functional relation among three dimensionless variables. With the sensible choice of dimensionless variables $\Pi_1 \equiv P/\rho g^{3/2} R^{5/2}$, $\Pi_2 \equiv \rho_s/\rho$ (the relative density of the bed), and $\Pi_3 \equiv d/R$ (the relative roughness of the bed), we may write $\Pi_1 = F[\Pi_2, \Pi_3]$ or, equivalently,

$$P = \rho g^{3/2} R^{5/2} F\left[\frac{d}{R}, \frac{\rho_s}{\rho}\right], \qquad (2)$$

where F is a dimensionless function of the relative density and of the relative roughness of the bed.

Note that in our dimensional analysis we have chosen P, the power of the jet, as a variable. This choice appears to be unprecedented; it places the focus of our analysis on the *energetics* of the turbulent cauldron; and it is the key to our results.

3.2 *Similarity*

To make further progress, we note that in applications $d/R \ll 1$, and seek to formulate an asymptotic similarity law for $d/R \to 0$.

In the case of complete similarity in d/R, $F[d/R, \rho_s/\rho]$ becomes independent of d/R as $d/R \to 0$. If this were the case, R would be independent of d for $d/R \ll 1$, which is incompatible with most of the empirical formulas that have been proposed so far, in which d appears explicitly (see Table 1). To obtain a theoretical formula that subsumes most empirical formulas proposed so far, we turn to the case of incomplete similarity in d/R.

In the case of incomplete similarity in d/R, (2) admits the following power-law asymptotic expression (Barenblatt 1996),

$$F\left[d/R, \rho_s/\rho\right] = (d/R)^{\alpha}\, G\left[\rho_s/\rho\right] + o\left[(d/R)^{\alpha}\right], \quad (3)$$

where α is a similarity exponent, which cannot be determined by dimensional analysis, and G is a dimensionless function of the relative density of the bed, ρ_s/ρ. By substituting the leading term of this asymptotic expression in (2) and rearranging, we obtain the following formula for R:

$$R = K q^{e_q} h^{e_h} g^{e_g} d^{e_d}\, H\left[\frac{\rho_s}{\rho}\right], \quad (4)$$

where $e_q = e_h = 2/(5-2\alpha)$, $e_g = -1/(5-2\alpha)$, $e_d = -2\alpha/(5-2\alpha)$, and we have defined $H[\Pi_2] \equiv 1/K(G[\Pi_2])^{2/(5-2\alpha)}$, where K is a dimensionless constant. The theoretical formula of Equation (4) has the same form as the generalized empirical formula of Equation (1) (provided that $H[\Pi_2] = 1/(\Pi_2 - 1)^{e_\rho}$). Nevertheless, the exponents that appear in both formulas (with the exception of e_ρ) are now revealed to be functions of a single free parameter, the similarity exponent. Because these functions were unknown, researchers developing empirical formulas treated the exponents of (1) as free parameters whose values had to be determined empirically (Table 1). A much improved way of determining these exponents suggests itself now, via the empirical determination of the similarity exponent. Yet we do not pursue this way of determining the exponents. Instead, in the following section we show that Equation (4) as well as the function $H[\rho_s/\rho]$ and the value of the similarity exponent can be derived in a completely independent way using the phenomenological theory of turbulence. Before turning to the next section we make two remarks, however.

First, note that the exponents in (4) are numbers: they do not depend on the variables of the problem. Formulas have been proposed in which the exponents depend on the variables of the problem, but these formulas lack a theoretical justification and are likely to be convoluted attempts at improving the fit of experimental data. Second, note that in the case of complete similarity in d/R (the case which we did not pursue), R would not depend on d, as we pointed out above, and the exponents of h and q would be $2/5$. Formulas have been proposed in which R does not depend on d, but in these formulas the empirical exponents of h and q do not appear to agree well with the theoretical value, $2/5$. We believe, in fact, that R does depend on d, and that we are justified in not pursuing the case of complete similarity in d/R.

4 DERIVATION OF THE FORMULA VIA THE PHENOMENOLOGICAL THEORY OF TURBULENCE

The phenomenological theory was originally derived for isotropic and homogeneous turbulent flows (Frisch 1998). But recent research (Knight & Sirovich 1990, Lundgren 2002, Lundgren 2003) indicates that the theory applies as well to flows that are neither isotropic nor homogeneous, as is the case of the flow in the turbulent cauldron.

The theory is based on two tenets pertaining to the steady production of turbulent (kinetic) energy: (1) The production occurs at the lengthscale of the largest eddies in the flow and (2) The rate of production is independent of the viscosity. From these tenets, it is possible to obtain a scaling expression for the rate of production of turbulent energy per unit mass of cauldron (which we denote by ε) in terms of the velocity of the largest eddies (which we denote by V) and of the size of the largest eddies (which scales with R). The largest eddies possess a kinetic energy per unit mass $e \sim V^2$ and a turnover time $t \sim R/V$, where '$\sim$' means 'scales with.' These eddies persist for a time t, whereupon they split into second-generation eddies (of size $\sim R/2$), thereby transferring their energy to smaller lengthscales. For the steady state to be preserved, a new set of large eddies must therefore be produced at time intervals t, implying that $\varepsilon = e/t \sim V^3/R$ (Lohse 1994). Now the second-generation eddies in turn split into third-generation eddies (of size $\sim R/4$), thereby transferring the kinetic energy they inherited from the second-generation eddies to even smaller lengthscales. In this manner, the kinetic energy is transfered to increasingly smaller lengthscales down to the Kolmogórov lengthscale, $\eta = \nu^{3/4}\varepsilon^{-1/4}$ (where ν is the kinematic viscosity), at which lengthscale the energy can be dissipated by the viscosity (Frisch 1998, Pope 2000). Thus, for a generation of eddies of size l and velocity u_l, it must be that $\varepsilon \sim u_l^3/l$, which together with $\varepsilon \sim V^3/R$ leads to the Kolmogórov scaling $u_l \sim V\,(l/R)^{1/3}$ (valid for $l/\eta \gg 1$). We recall these results later on.

Now we consider the energetics of the turbulent cauldron and seek to obtain a scaling expression for V, the velocity of the largest eddies. The production of turbulent energy is driven by the jet, whose power per unit thickness is $P = q\rho gh$. Therefore, P must equal the rate of production of turbulent energy per unit thickness of cauldron (note that P is independent of the viscosity, in accord with the second tenet of the phenomenological theory stated above), and we can write $P = \varepsilon M$, where ε is the rate of production of turbulent energy per unit mass, and $M \sim \rho R^2$ is the mass per unit thickness of cauldron. It follows that $\varepsilon \sim qgh/R^2$ and, from a comparison with $\varepsilon \sim V^3/R$, that

$$V \sim \left(qg\frac{h}{R}\right)^{1/3}, \tag{5}$$

which is the sought expression for the velocity of the largest eddies in the cauldron.

Next we consider the surface of the pothole and seek to obtain a scaling expression for the shear stress exerted by the flow on that surface. We start with a brief outline of our assumptions regarding the structure of the turbulent flow close to the surface of the pothole.

There is a vast literature on the structure of the turbulent flow close to smooth and rough walls. Depending on the author and the phenomena of interest, there may be more or fewer turbulent layers, and a number of differing criteria have been proposed as to what defines a turbulent layer. Nevertheless, there is no specific turbulent structure that is accepted by everyone and that is useful regardless of the phenomena of interest. In his very recent review, Jiménez has pointed out that the "turbulent structure" close to a wall "is far from being understood. There are conflicting experiments in almost all cases and, even for those quantities for which the trends are clear, the data collapse is poor" (Jiménez 2004). We infer that the actual structure of the turbulent flow close to a wall has remained largely inaccessible via experiments; it is thus hardly possible to validate any proposed structure by means of a *direct* comparison with the actual structure of the flow. In fact, any proposed structure is a theoretical construct that must be ultimately judged *indirectly*, by its capacity to yield predictions that match measurements of experimentally accessible quantities. Give this state of affairs, we adopt a simple structure: we assume that there exists a single turbulent layer governed by the phenomenological spectrum. This simple single-layer structure has allowed us to predict the scalings of Blasius and Strickler, for smooth and rough walls, respectively (Gioia and Bombardelli 2002).

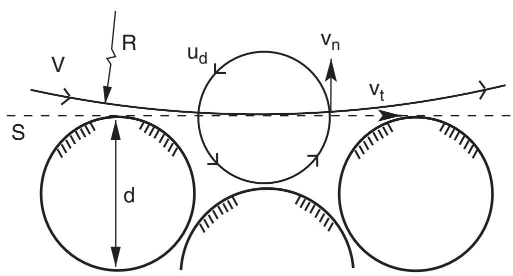

Figure 2. Three grains of diameter d lying at the surface of the pothole. The dashed line is the trace of a wetted surface S tangent to the peaks of the grains at the surface of the pothole.

Let us call S a wetted surface tangent to the peaks of the grains at the surface of the pothole (Figure 2). Under conditions of fully developed turbulence, the shear stress acting on S is the Reynolds stress, $\tau = \rho|\overline{v_n v_t}|$, where v_n and v_t are the fluctuating velocities normal and tangent to S, respectively, and an overbar denotes time average. We study v_n first, and start by making a crucial observation: if the relative roughness is small ($d/R \ll 1$), eddies of sizes larger than, say, $2d$, can make only a negligible contribution to v_n (this is entirely a matter of geometry; see Figure 2). On the other hand, eddies smaller than d fit in the space between successive grains on the bed, so that these eddies can make a sizable contribution to v_n. Nevertheless, where these eddies are smaller than, say, $d/2$, their velocities are negligible compared with the velocity of the eddies of size d. (Recall that according to the Kolmogórov scaling, $u_l \sim V(l/R)^{1/3}$ (valid for $l/\eta \gg 1$); therefore, the smaller the size of an eddy, l, the smaller its velocity, u_l.) Thus, assuming that $d/\eta \gg 1$, v_n is dominated by u_d, the velocity of the eddies of size d. In other words, $v_n \sim u_d$. We now turn to v_t. By virtue of simple geometrical considerations, eddies of all sizes can provide a velocity tangent to S. Thus, v_t is dominated by V, the velocity of the largest eddies, and $v_t \sim V$. We conclude that $|\overline{v_n v_t}| \sim u_d V$, and therefore $\tau \sim \rho u_d V$ (Gioia and Bombardelli 2002).

From our scaling for the shear stress, $\tau \sim \rho u_d V$, we conclude that the horizontal and vertical components of the velocity fluctuations scale differently. This conclusion is supported by recent experimental measurements (B. Hoftland, J. Battjes, R. Booij, 2005, in press, J. Hyd. Eng.). Note also that the shear stress peaks at the bottom of the pothole (see Beltaos 1976, in particular Figure 9) and remains practically constant over broad bands of the bed on either side of the bottom of the pothole.

We may now substitute (5) and $u_d \sim V(d/R)^{1/3}$ in $\tau \sim \rho u_d V$ to obtain

$$\tau \sim \rho \frac{(qhg)^{2/3}\, d^{1/3}}{R}, \tag{6}$$

which is the sought expression for the shear stress exerted by the turbulent cauldron on the surface of the pothole, valid for $\eta \ll d \ll R$. To discuss Equation (6), it is convenient to rewrite it in terms of the power of the jet per unit thickness, $P = q\rho gh$, with the result $\tau \sim P^{2/3}(\rho d)^{1/3}/R$. Consider now the instant when the jet of power P plunges into the pool of water of uniform depth D. Then, the pothole starts to form, and as the depth Δ of the pothole increases, the size $R = \Delta + D$ of the cauldron increases accordingly, leading to a decrease in the shear stress on the surface of the pothole. Eventually, the shear stress decreases to a critical value τ_c, and the scouring ceases. Thus the condition of equilibrium between the turbulent cauldron and the granular bed is $\tau = \tau_c$ (Yalin 1977, Raudkivi 1999).

To obtain a scaling expression for the critical stress τ_c, we follow Shields (Yalin 1977) in recognizing that the grains at the surface of a granular bed are subjected to a Reynolds stress $\tau \sim \rho u_d V$ (exerted by the turbulent flow), a gravitational stress $\tau_g \sim (\rho_s - \rho)gd$, and a viscous stress $\tau_\nu \sim \rho\nu V/d$. Then, if the equilibrium condition is satisfied, we can perform a straightforward dimensional analysis using three variables: $\tau = \tau_c$, τ_g, and τ_ν. The result is $\tau_c \sim \tau_g\, I[\mathrm{Re}_d]$, where I is a dimensionless function of a Reynolds number $\mathrm{Re}_d \equiv \tau/\tau_\nu = u_d\, d/\nu$. By recalling that $\varepsilon \sim u_d^3/d$, $\eta = \nu^{3/4}\varepsilon^{-1/4}$, and $d/\eta \gg 1$, we conclude that $\mathrm{Re}_d \sim (d/\eta)^{4/3} \gg 1$, and seek to formulate a similarity law for $\mathrm{Re}_d \to \infty$. If we assume complete similarity in Re_d, then $I[\mathrm{Re}_d]$ tends to a constant as $\mathrm{Re}_d \to \infty$ (as indicated by experimental results on incipient motion of granular beds (Yalin 1977), and therefore $\tau_c \sim (\rho_s - \rho)gd$, which is the sought expression for the critical stress.

We are now ready to impose the equilibrium condition. By substituting (6) and $\tau_c \sim (\rho_s - \rho)gd$ into $\tau = \tau_c$ and rearranging, we obtain the following scaling expression for R:

$$R \sim q^{2/3} h^{2/3} g^{-1/3} d^{-2/3} \left(\frac{\rho}{\rho_s - \rho}\right). \tag{7}$$

This expression gives (up to a multiplicative constant) a complete theoretical formula to compute the equilibrium depth of the pothole as $\Delta = R - D$, where D is the depth of the pool of water (Figure 1). A comparison of (7) with (4) indicates that $e_q = e_h = 2/3$, $e_g = -1/3$, and $e_d = -2/3$, in accord with a similarity exponent of value $\alpha = 1$. Thus, the theory gives values of e_q, e_h, e_g, and e_d that relate to one another in the form necessitated by the independent analysis that yielded (4). Further, a comparison of (7) with (4) indicates that $H[\Pi_2] = 1/(\Pi_2 - 1)$, which corresponds to the generalized empirical formula of Equation (1) with $e_\rho = 1$.

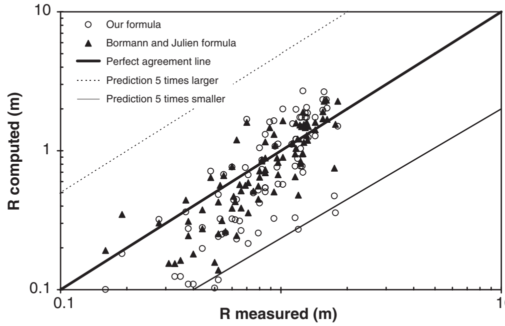

Figure 3. Plot of the measured value of R versus the predicted value of R for our theoretical exponents and for the semi-empirical exponents of Bormann and Julien. The experimental data are those of Bormann and Julien (1991).

5 VALIDATION OF THE FORMULA

The theoretical values of the exponents obtained in the previous section appear in Table 1, where they may be compared with the corresponding empirical values determined by various researchers. This comparison affords a considerable degree of experimental support to our theoretical results, even though for each exponent the alternative empirical values vary over a sizable range, as might have been expected where many different, difficult experiments were involved.

We test the theory further by comparing our predictions directly with experimental data. Figure 3 shows a plot of the measured values of R versus the predicted values of R for our theoretical exponents and for the semi-empirical exponents of Bormann and Julien—the exponents that appear to be the most similar to our theoretical exponents. The experimental data of Fig. 3 are those of Bormann and Julien; therefore, these are the very data that Bormann and Julien fitted in order to determine the values of their semi-empirical exponents. Yet our theoretical exponents attain almost an equally good fit.

6 CONCLUSIONS

In this paper, we have focused on the energetics of the turbulent cauldron to derive (up to a multiplicative constant) a theoretical formula for the depth of a pothole in equilibrium with a turbulent cauldron driven by a jet of constant power. The formula represents the power-law asymptotic behavior of a fully developed turbulent flow of incomplete similarity in the relative roughness of the cohesionless granular bed. We have validated our predictions through a comparison of our theoretical exponents with the exponents of several empirical formulas. We have also compared our predictions directly with experimental data.

In deriving the formula based on the phenomenological theory, we have gained insights into the form of interaction between the cauldron and the granular bed. These insights are of obvious theoretical import, but they also suggest improved ways of dealing with applications. Thus, for example, our discussion of the equilibrium condition between the bed and the cauldron, $\tau = \tau_c$, suggests that to incorporate a *physically meaningful* safety factor in the design of a pothole we might impose the *design* condition $\tau = \tau_c/f$, where $f > 1$ is the safety factor. The design value of the depth of the pothole would then be $\Delta_{\text{design}} = f\,\Delta + (f-1)D$, where Δ is the equilibrium value of the depth.

In conclusion, our results indicate that despite current practice, theory may be advantageously used instead of empirical formulas in the analysis and design of overflowing gates, weirs, dams, and natural obstructions.

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